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n-ABSORBING I-IDEALS

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ABSTRACT. Let R be a commutative ring with identity, let I be a proper ideal of R, and let $n \geq 1$ be a positive integer. In this paper, we introduce a class of ideals that is closely related to the class of I-prime ideals. A proper ideal P of R is called an *n-absorbing I-ideal* if $a_1,\ a_2,\ \ldots,\ a_{n+1} \in R$ with $a_1 a_2 \ldots a_{n+1} \in P - IP$, then $a_1 a_2 \ldots a_{i-1} a_{i+1} \ldots a_{n+1} \in P$ for some $i \in \{1,\ 2,\ \ldots,\ n+1\}$. Among many results, we show that every proper ideal of a ring R is an *n-absorbing I-ideal* if and only if every quotient of R is a product of $(n+1)$-fields.

1. Introduction

Throughout this article, R denotes a commutative ring with identity and $Max(R)$ denotes the set of all maximal ideals of R. The notion of *prime* ideal plays a main role in the theory of commutative algebra and it has been widely studied and recently many generalizations were introduced by many authors. Recall from [4] that a *prime* ideal of R is a proper ideal P with the property that for $a, b \in R$, $ab \in P$ implies $a \in P$ or $b \in P$. The concept of *I-prime* ideals was defined and investigated in [1]. For a fixed ideal I of R, a proper ideal P of R is *I-prime* if $a, b \in R$ with $ab \in P - IP$ implies either $a \in P$ or $b \in P$. The concept of *2-absorbing* ideals was introduced and studied in [5]. Let n be a positive integer. A proper ideal P of a ring R is called an *n-absorbing* ideal if whenever $x_1 \ldots x_{n+1} \in P$ for $x_1, \ldots, x_{n+1} \in R$, then there are n of the x_i's whose product is in P. Equivalently, a proper ideal P of R is an *n-absorbing* ideal if and only if whenever $x_1 \cdots x_m \in P$ for $x_1, \ldots, x_m \in R$ with $m > n$, then there are n of the x_i's whose product is in P; see [3].

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Let $n \geq 2$ and let $\phi : S(R) \to S(R) \cup \{\phi\}$ be a map, where $S(R)$ is the set of ideals of R. A proper ideal P of R is called $(n-1, n)$-*ϕ-prime* if whenever $a_1, a_2, \ldots, a_n \in R$ with $a_1 a_2 \ldots a_n \in P - \phi(P)$, then the product of $n-1$ of the a_i's is in P (see [6]).

In this article, we introduce a class of ideals that is closely related to the class of *I-Prime* ideals. Let I be a proper ideal of R and let $n \geq 1$. A proper ideal P of R is called an *n-absorbing I-prime* ideal of R if $a_1, a_2, \ldots, a_{n+1} \in R$ with $a_1 a_2 \ldots a_{n+1} \in P - IP$, then $a_1 a_2 \ldots a_{i-1} a_{i+1} \ldots a_{n+1} \in P$ for some $i \in \{1, 2, \ldots, n+1\}$. Thus a *1-absorbing I-ideal* is just an *I-prime* ideal. If we set $\phi(P) = IP$ for every P in $S(R)$, then the ideas of this paper are a special case of the paper [6]. Some properties of the *n-absorbing I-prime* ideals are discussed and studied.

2. Main results

Let I be a fixed ideal of a ring R and let $n \geq 1$ be a positive integer. A proper ideal P of R is called an *n-absorbing I-ideal* if $a_1, a_2, \ldots, a_{n+1} \in R$ with $a_1 \ldots a_{n+1} \in P - IP$, then $a_1 \ldots a_{i-1} a_{i+1} \ldots a_{n+1} \in P$ for some $i \in \{1, 2, \ldots, n+1\}$. It is clear that a proper ideal P is an *n-absorbing I-ideal* of R if and only if whenever the product of $(n+1)$-elements of R/P is 0, then the product of some n of these elements is 0 in R/P. Not all *n-absorbing I-ideals* are $(n-1)$-*absorbing I-ideal*. The following example illustrates this fact.

Example 2.1. Consider the ring $R = \dfrac{k[[x,y]]}{\langle x^n, y^n, x^{2n} - y^{2n}, x^{2n+1} y^{2n++1} \rangle}$, where k is a field and $n \geq 1$ is a positive integer. Put the fixed ideal I to be zero ideal of R. Then the proper ideal $P = \langle \bar{x}^n, \bar{y}^n, \bar{x}^{2n} - \bar{y}^{2n}, \bar{x}^{2n+1} \bar{y}^{2n+1} \rangle$ of R is a $(2n+1)$-*absorbing I-ideal* but not a $2n$-*absorbing I-ideal*, since $\bar{x}^{2n} \in P$ and $\bar{x}^{2n-1} \notin P$.

The proof of the following lemma comes directly from the definition so it is omitted.

Lemma 2.2. *A proper ideal P of a ring R is an n-absorbing I-ideal if and only if $\dfrac{P}{IP}$ is an n-absorbing 0-ideal.*

Proposition 2.3. *Let P be an n-absorbing I-ideal of a ring R and let $S \subseteq R$ be a multiplicative closed set of R such that $P \cap S = \phi$. Then $S^{-1}P$ is an n-absorbing $S^{-1}I$-ideal of $S^{-1}R$.*

Proof. Suppose $\frac{a_1}{s_1}, \ldots, \frac{a_{n+1}}{s_{n+1}} \in S^{-1}R$ with $\frac{a_1 a_2 \ldots a_{n+1}}{s_1 s_2 \ldots s_{n+1}} \in S^{-1}P - S^{-1}IS^{-1}P = S^{-1}(P - IP)$. Then $ua_1 a_2 \ldots a_{n+1} \in P_I P$ for some $u \in S$. By taking ua_1 as one element, either $a_2 \ldots a_{n+1} \in P$ or $ua_1 \cdots a_{i-1} a_{i+1} \cdots a_{n+1} \in P$ for $i = 2, 3, \ldots, n+1$. Hence $\frac{a_2 \ldots a_{n+1}}{s_2 \ldots s_{n+1}} = \frac{a_2}{s_2} \cdots \frac{a_{n+1}}{s_{n+1}} \in S^{-1}P$ or $\frac{ua_1 \ldots a_{i-1} a_{i+1} \ldots a_{n+1}}{us_1 \ldots s_{i-1} s_{i+1} \ldots s_n} = \frac{a_1}{s_1} \cdots \frac{a_{i-1}}{s_{i-1}} \frac{a_{i+1}}{s_{i+1}} \cdots \frac{a_{n+1}}{s_{n+1}} \in S^{-1}P$, which means that $S^{-1}P$ is an *n-absorbing* $S^{-1}I$-*ideal* of $S^{-1}R$. $\square$

Theorem 2.4. *Let P be a proper ideal of a commutative ring R. If P is an n-absorbing I-ideal that is not an n-absorbing ideal, then $P^{n+1} \subseteq IP$.*

Proof. Assume that $P^n \nsubseteq IP$. We have to show that P is an n-*absorbing* ideal. Let $x_1x_2\ldots x_{n+1} \in P$ for $x_1,\ x_2,\ \ldots,\ x_{n+1} \in R$. If $x_1x_2\ldots x_{n+1} \notin IP$, then the n-*absorbing* I-*ideal* P gives that P is an n-*absorbing* ideal. Now, for the case $x_1x_2\ldots x_{n+1} \in IP$, we have $x_1x_2\ldots x_{n+1-k}P^k \subseteq IP$ for $k = 1,\ 2,\ \ldots,\ n$, since otherwise, we obtain $x_1x_2\ldots x_{n+1-k}p_1p_2\ldots p_k \notin IP$ for $p_1,\ p_2,\ \ldots,\ p_k \in P$ and so $x_1x_2\ldots x_{n+1-k}\,(x_{n+2-k}+p_1)\ldots(x_{n+1}+p_k) \in P - IP$. As P is an n-*absorbing* I-*ideal*, $x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_{n+1} \in P$, for some $i = \{1,\ 2,\ \ldots,\ n+1\}$. Similarly, we can assume that for all $i_1, i_2, \ldots, i_{n+1-k} \subseteq \{1,\ 2,\ \ldots,\ n+1\}$, $a_{i_1}\ldots a_{i_{n+1-k}}P^k \subseteq IP$ with $1 \leq k \leq n+1$. Since $P^{n+1} \nsubseteq IP$, there exist $r_1, r_2, \ldots, r_{n+1} \in P$ with $r_1r_2\ldots r_{n+1} \notin IP$. Then $(x_1+r_1)(x_2+r_2)\ldots(x_{n+1}+r_{n+1}) \in P - IP$. Thus being P n-*absorbing* I-*ideal* gives us $x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_{n+1} \in P$ for some $i \in \{1,\ 2,\ \ldots,\ n+1\}$. Therefore P is an n-*absorbing* ideal.

□

We conclude from Theorem 2.4 that an n-*absorbing* I-*ideal* P with $P^{n+1} \nsubseteq IP$ is an n-*absorbing* ideal.

Corollary 2.5. *Let R be a ring and let P be a proper ideal of R. If P is an n-absorbing 0-ideal that is not an n-absorbing ideal, then $P^{n+1} = 0$.*

Corollary 2.6. *Let P be an n-absorbing I-ideal with $(IP) \subseteq P^{n+2}$. Then P is an n-absorbing $\cap_{i=1}^{\infty}P^i-$ideal $(n \geq 1)$.*

Proof. If P is an n-*absorbing* ideal, then P is an n-*absorbing* I-*ideal* and so is an n-*absorbing* $\cap_{i=1}^{\infty}P^i$-*ideal*. Suppose that P is not an n-*absorbing* ideal, then Theorem 2.4 gives us $P^{n+1} \subseteq IP \subseteq P^{n+2}$. Hence $IP = P^k$ for each $k \geq n+1$ and hence $\cap_{i=1}^{\infty}P^i = IP$. Thus P is an n-*absorbing* $\cap_{i=1}^{\infty}P^i$-*ideal*. □

Let R and S be two rings. If P is an n-*absorbing* 0-*ideal* of R. Then $P \times S$ need not be an n-*absorbing* 0-*ideal* of $R \times S$. For a particularly case see [2, Theorem 7]. However, $P \times S$ is an n-*absorbing* I-*ideal* for each I with $\cap_{i=1}^{\infty}(P\times S)^i \subseteq I(P\times S) \subseteq P\times S$.

Theorem 2.7. (1) *Let R and S be two rings and let P be an n-absorbing 0-ideal of R. Then $J = P \times S$ is an n-absorbing I-ideal of $R \times S$, for each I with $\cap_{i=1}^{\infty}(P\times S)^i \subseteq I(P\times S) \subseteq P\times S$.*

(2) *Let R be a commutative ring and let J be a finitely generated proper ideal of R. Suppose that J is an n-absorbing I-ideal, where $IP \subseteq J^{n+2}$. Then either J is an n-absorbing 0-ideal or $J^{n+1} \neq 0$ is idempotent and R decomposes as $T \times S$, where $S = J^{n+1}$ and $J = P \times S$, where P is an n-absorbing 0-ideal. Hence J is an n-absorbing I-ideal for each I with $\cap_{i=1}^{\infty}J^i \subseteq IJ \subseteq J$.*

Proof. (1) Let R and S be two rings and let P be an n-*absorbing* 0-*ideal* of R. Then $P \times S$ need not be an n-*absorbing* 0-*ideal* of $R \times S$. In fact, $P \times S$ is an n-*absorbing* 0-*ideal* if and only if $P \times S$ is a *prime* ideal. However, $P \times S$ is an n-*absorbing* I-*ideal* for each I with $\cap_{i=1}^{\infty}(P\times S)^i \subseteq I(P\times S)$. If P is an n-*absorbing* ideal, then $P \times S$ is an n-*absorbing* ideal and thus is an n-*absorbing* I-*ideal*. Assume that P is not an n-*absorbing* ideal. Then $P^{n+1} = 0$ and $(P\times S)^{n+1} = 0 \times S$. Hence $\cap_{i=1}^{\infty}(P\times S)^i = \cap_{i=1}^{\infty}P^i \times S = 0 \times S$. Thus $P \times S - \cap_{i=1}^{\infty}(P\times S)^i =$

$P \times S - 0 \times S = (P - 0) \times S$. Since P is an n-*absorbing* 0-*ideal*, $P \times S$ is an n-*absorbing* $\cap_{i=1}^{\infty}(P \times S)^i$-*ideal* and as $\cap_{i=1}^{\infty}(P \times S)^i \subseteq I(P \times S)$, $P \times S$ is an n-*absorbing* I-*ideal*.

(2) If J is an n-*absorbing* ideal, then J is an n-*absorbing* 0-*ideal*. So, we can assume that J is not an n-*absorbing* ideal. Then $J^{n+1} \subseteq IP$ and hence $J^{n+1} \subseteq IP \subseteq J^{n+2}$, so $J^{n+1} = J^{n+2}$. Hence J^{n+1} is idempotent. Since J^{n+1} is finitely generated, $J^{n+1} = (e)$ for some idempotent $e \in R$. Suppose $J^{n+1} = 0$. Then $IP = 0$, and hence J is an n-*absorbing* 0-*ideal*. Assume that $J^{n+1} \neq 0$, and put $S = J^{n+1} = Re$ and $T = R(1-e)$, so R decomposes $T \times S$. Let $P = J(1-e)$; so $J = P \times S$, where $P^{n+1} = (J(1-e))^{n+1} = J^{n+1}(1-e)^{n+1} = (e)(1-e) = 0$. We claim that P is an n-*absorbing* 0-*ideal*. Let $x_1, x_2, \ldots, x_{n+1} \in R$ and let $0 \neq x_1x_2\ldots x_{n+1} \in P$. Then $(x_1, 0)(x_2, 0)\ldots(x_{n+1}, 0) = (x_1x_2\ldots x_{n+1}, 0) \in P \times S - (P \times S)^{n+1} = P \times S - 0 \times S \subseteq P - IP$, since $IP \subseteq J^{n+2}$, which implies that $IP \subseteq J^{n+2} = (P \times S)^{n+2} = 0 \times S$. Hence $J - J^{n+1} \subseteq J - IP$. As J is an n-*absorbing* I-*ideal*, $(x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_{n+1}, 0) \in P \times S = J$, for some $i = \{1, 2, \ldots, n+1\}$. Thus $x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_{n+1} \in P$. Hence P is an n-*absorbing* 0-*ideal*. □

Corollary 2.8. *Let R be an indecomposable ring and let P be a finitely generated n-absorbing I-ideal of R, where $IP \subseteq P^{n+2}$. Then P is an n-absorbing 0-ideal. Furthermore, if R is an integral domain, then P is actually an n-absorbing ideal.*

Corollary 2.9. *A proper ideal P of a Noetherian integral domain R is an n-absorbing ideal if and only if P is an n-absorbing P^{n+1}-ideal for $(n \geq 2)$.*

In what follows, we characterize an n-*absorbing* I-*ideals*.

Theorem 2.10. *Let P be a proper ideal of a ring R. Then the following conditions are equivalent.*

(1) *P is an n-absorbing I-ideal.*

(2) *For $x_1, x_2, \ldots, x_n \in R - P$:*

$$(P: x_1x_2\ldots x_n) = \cup_{i=1}^{n}(P : x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_n) \cup (IP : x_1x_2\ldots x_n).$$

Proof. (1) $\Rightarrow$ (2) Suppose $x_1, x_2, \ldots, x_n \in R - P$ and $y \in (P : x_1x_2\ldots x_n)$. Then $x_1x_2\ldots x_n y \in P$. If $x_1x_2\ldots x_n y \notin IP$, then $x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_n y \in P$, for some $i \in \{1, 2, \ldots, n\}$, and so $y \in (P : x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_n)$. If $x_1x_2\ldots x_n y \in IP$, then $y \in (IP : x_1x_2\ldots x_n)$. Hence

$$(P: x_1x_2\ldots x_n) \subseteq \cup_{i=1}^{n}(P : x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_n) \cup (IP : x_1x_2\ldots x_n).$$

The other containment always holds.

(2) $\Rightarrow$ (1) Suppose $x_1x_2\ldots x_{n+1} \in P - IP$. If $x_1x_2\ldots x_n \in P$, then there is nothing to prove. Assume that $x_1x_2\ldots x_n \notin P$. Thus

$$(P: x_1x_2\ldots x_n) = \cup_{i=1}^{n}(P : x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_n) \cup (IP : x_1x_2\ldots x_n).$$

Since $x_1x_2\ldots x_{n+1} \in P$, $x_{n+1} \in (P: x_1x_2\ldots x_n)$ and the fact $x_1x_2\ldots x_{n+1} \notin IP$ gives us $x_{n+1} \notin (IP: x_1x_2\ldots x_n)$. Hence $x_{n+1} \in (P : x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_n)$, for some $i \in \{1, 2, \ldots, n\}$, that is, $x_1x_2\ldots x_{i-1}x_{i+1}\ldots x_{n+1} \in P$. Thus P is an n-*absorbing* I-*ideal*. □

It was shown by Anderson and Smith [2, Theorem 8] that every proper ideal of R is weakly *prime* if and only if R is a direct product of two fields or (R, m) is quasi-local with $M^2 = 0$. Next we generalize this result to an *n-absorbing I-ideals* but first we need the following lemma.

Lemma 2.11. *Let $R = R_1 \times R_2 \times \cdots \times R_{n+1}$, where R_i is a ring, for $i \in \{1, 2, \ldots, n+1\}$. If P is an n-absorbing I-ideal of R, then either $P = IP$ or $P = P_1 \times P_2 \times \cdots \times P_{i-1} \times R_i \times P_{i+1} \times \cdots \times P_{n+1}$ for some $i \in \{1, 2, \ldots, n+1\}$ and if $P_j \neq R_i$ for $j \neq j$, then P_j is an n-absorbing ideal in R_j.*

Proof. Let $P = P_1 \times P_2 \times \cdots \times P_{n+1}$ be an *n-absorbing I-ideal* of R. Then there exists $(x_1, x_2, \ldots, x_{n+1}) \in P - IP$, and so

$$(x_1, 1, \ldots, 1)(1, x_2, 1, \ldots, 1) \ldots (1, 1, \ldots, x_{n+1}) = (x_1, x_2, \ldots, x_{n+1}) \in P.$$

As P is an *n-absorbing I-ideal*, we have $(x_1, x_2, \ldots, x_{i-1}, 1, x_{i+1}, \ldots, x_{n+1}) \in P$ for some $i \in \{1, 2, \ldots, n+1\}$. Thus $(0, 0, \ldots, 0, 1, 0, \ldots, 0) \in P$ and hence $P = P_1 \times P_2 \times \cdots \times P_{i-1} \times R_i \times P_{i+1} \times \cdots \times P_{n+1}$. If $P_j \neq R_j$ for $j \neq i$, then we have to prove P_j is an *n-absorbing* ideal in R_j. Let $i < j$ and let $y_1 y_2 \ldots y_{n+1} \in P_j$. Then

$$\begin{aligned} &(0, 0, \ldots, 0, 1, 0, \ldots, 0, y_1 y_2 \ldots y_n, 0, \ldots, 0) \\ &\quad = (0, 0, \ldots, 1, 0, \ldots, y_1, \ldots, 0)(0, 0, \ldots, 1, 0, \ldots, y_2, \ldots, 0) \\ &\qquad \ldots (0, 0, \ldots, 1, 0, \ldots, y_{n+1}, \ldots, 0) \in P - IP \end{aligned}$$

and the *n-absorbing I-ideal* P give that

$$(0, 0, \ldots, 0, 1, 0, \ldots, 0, y_1 y_2 \ldots y_{k-1} y_{k+1} \ldots y_{n+1}, 0, \ldots, 0) \in P$$

for some $k \in \{1, 2, \ldots, n+1\}$. Thus $y_1 y_2 \ldots y_{k-1} y_{k+1} \ldots y_{n+1} \in P_j$ and hence P_j is an *n-absorbing* ideal in R_j. We can do the same arguments for the case $j < i$. $\square$

Theorem 2.12. *Let R be a ring and let $|Max(R)| \geq n + 1 \geq 2$. Every proper ideal of R is an n-absorbing I-ideal if and only if every quotient of R is a product of $(n+1)$-fields.*

Proof. ($\Leftarrow$): Let P be a proper ideal of R. Then $\frac{R}{IP} \cong F_1 \times F_2 \times \cdots \times F_{n+1}$ and $\frac{P}{IP} \cong P_1 \times P_2 \times \cdots \times P_{n+1}$, where P_i is an ideal of F_i, $i = 1, 2, \ldots, n+1$. If $P = IP$, then there is nothing to prove, otherwise we have $P_j = 0$ for at least one $j \in \{1, 2, \ldots, n+1\}$, since $\frac{P}{IP}$ is proper. Therefore $\frac{P}{IP}$ is an *n-absorbing 0-ideal* of $\frac{R}{IP}$ and P is an *n-absorbing I-ideal* of R.

($\Rightarrow$): Let $m_1, m_2, \ldots, m_{n+1}$ be distinct maximal ideals of R. Then $m = m_1 m_2 \ldots\ m_{n+1}$ is an *n-absorbing I-ideal* of R. We want to show that m is not an *n-absorbing* ideal. First to show that $m_i \nsubseteq \cup_{j \neq i} m_j$ for all $i \in \{1, 2, \ldots, n+1\}$, we suppose the contrary that $m_i \subseteq \cup_{j \neq i} m_j$. Then there exists m_j with $m_i \subseteq m_j$ by *prime* avoidance lemma, which contradicts the fact that m_i, $i = 1, 2, \ldots, n+1$ are distinct maximal ideals. Hence there exists $x_i \in m_i - \cup_{i \neq j = 1}^{n+1} m_j$ and so $x_1, x_2, \ldots, x_{n+1} \in m$. If there exists $j \in \{1, 2, \ldots, n+1\}$ with
$x_1 x_2 \ldots x_{j-1} x_{j+1} \ldots x_{n+1} \in m \subseteq m_j$, then $x_i \in m_j$, for some $i \neq j$, a contradiction. Hence m is not an *n-absorbing* ideal and so $m^{n+1} = Im$. Thus by the

Chinese remainder theorem, $\frac{R}{Im} \cong \frac{R}{m_1^{n+1}} \times \frac{R}{m_2^{n+1}} \times \cdots \times \frac{R}{m_{n+1}^{n+1}}$. Put $F_i = \frac{R}{m_i^{n+1}}$. If F_i is not field, then it has a nonzero proper ideal K and so $0 \times 0 \times \cdots \times 0 \times K \times 0 \times \cdots \times 0$ is an *n-absorbing* 0*-ideal* of $\frac{R}{Im}$. Thus by Lemma 2.11, we have $K = F_i$ or $K = 0$, which is impossible. Hence F_i is a field. □

Corollary 2.13. *Let R be a ring and let $|Max(R)| \geq n+1 \geq 2$. Every proper ideal of R is an n-absorbing 0-ideal if and only if $R \cong F_1 \times F_2 \times \cdots \times F_{n+1}$, where $F_1, F_2, \ldots, F_{n+1}$ are fields.*

In what follows, we characterize rings with the property that every proper ideal is an *n-absorbing* 0*-ideal*.

Corollary 2.14. *Let P be an n-absorbing I-ideal of a ring R, where $IP \subseteq P^{n+2}$. Then P is an n-absorbing $\cap_{i=1}^{\infty} P^i$-ideal $(n \geq 2)$.*

Proof. If P be an *n-absorbing* ideal, then P is an *n-absorbing I-ideal* and so is an *n-absorbing* $\cap_{i=1}^{\infty} P^i$*-ideal*. Suppose that P is not an *n-absorbing* ideal. Then Theorem 2.4 gives us $P^{n+1} \subseteq IP \subseteq P^{n+2}$. Hence $IP = P^k$ for each $k \geq n+1$ and hence $\cap_{i=1}^{\infty} P^i = IP$. Thus P is an *n-absorbing* $\cap_{i=1}^{\infty} P^i$*-ideal*. □

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