Khayyam J. Math. 4 (2018), no. 1, 39–58

DOI: 10.22034/kjm.2017.54680

Khayyam Journal of Mathematics

emis.de/journals/KJM
kjm-math.org

OSTROWSKI TYPE FRACTIONAL INTEGRAL OPERATORS FOR GENERALIZED BETA (r, g)-PREINVEX FUNCTIONS

ARTION KASHURI[1]*, ROZANA LIKO[1] AND TINGSONG DU[2]

Communicated by M. Ito

Abstract. In the present paper, the notion of generalized beta (r, g)-preinvex function is applied for establish some new generalizations of Ostrowski type inequalities via fractional integral operators. These results not only extend the results appeared in the literature [43] but also provide new estimates on these type. At the end, some applications to special means are given.

1. Introduction

The following notations are used throughout this paper. We use I to denote an interval on the real line $\mathbb{R} = (-\infty, +\infty)$ and I° to denote the interior of I. For any subset $K \subseteq \mathbb{R}^n$, K° is used to denote the interior of K. $\mathbb{R}^n$ is used to denote an n-dimensional vector space. The set of integrable functions on the interval $[a, b]$ is denoted by $L_1[a, b]$.

The following result is known in the literature as the Ostrowski inequality [26] and the references cited therein, which gives an upper bound for the approximation of the integral average $\dfrac{1}{b-a}\displaystyle\int_a^b f(t)dt$ by the value $f(x)$ at point $x \in [a, b]$.

Date: Received: 6 June 2017; Revised: 25 December 2017; Accepted: 30 December 2017.

* Corresponding author.

2010 *Mathematics Subject Classification.* Primary 26A51; Secondary 26A33, 26D07, 26D10, 26D15.

Key words and phrases. Ostrowski type inequality, Hölder's inequality, Minkowski's inequality, power mean inequality, Riemann-Liouville fractional integral, fractional integral operator, s-convex function in the second sense, m-invex.

Theorem 1.1. *Let $f : I \longrightarrow \mathbb{R}$, where $I \subseteq \mathbb{R}$, be a mapping differentiable in I° and let $a, b \in I^\circ$ with $a < b$. If $|f'(x)| \leq M$ for all $x \in [a, b]$, then*

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(t)dt \right| \leq M(b-a)\left[\frac{1}{4} + \frac{\left(x - \frac{a+b}{2}\right)^2}{(b-a)^2}\right], \quad \forall x \in [a, b]. \tag{1.1}$$

For other recent results concerning Ostrowski type inequalities, see [2]-[6],[8], [10]-[13],[15],[17]-[19],[21],[24],[25],[28]-[30],[33]-[35],[37],[38],[40],[41],[43],[44], and the references cited therein. Ostrowski inequality is playing a very important role in all the fields of mathematics, especially in the theory of approximations. Thus such inequalities were studied extensively by many researches and numerous generalizations, extensions and variants of them for various kind of functions like bounded variation, synchronous, Lipschitzian, monotonic, absolutely, continuous and n-times differentiable mappings etc. appeared in a number of papers, see [10],[11],[13]. In recent years, one more dimension has been added to this studies, by introducing a number of integral inequalities involving various fractional operators like Riemann-Liouville, Erdelyi-Kober, Katugampola, conformable fractional integral operators etc. by many authors, see [1],[22],[23],[32],[38],[39]. Riemann-Liouville fractional integral operators are the most central between these fractional operators.

Fractional calculus, see [27] and the references cited therein, was introduced at the end of the nineteenth century by Liouville and Riemann, the subject of which has become a rapidly growing area and has found applications in diverse fields ranging from physical sciences and engineering to biological sciences and economics.

Definition 1.2. Let $f \in L_1[a, b]$. The Riemann-Liouville integrals $J^\alpha_{a+}f$ and $J^\alpha_{b-}f$ of order $\alpha > 0$ with $a \geq 0$ are defined by

$$J^\alpha_{a+}f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1}f(t)dt, \quad x > a$$

and

$$J^\alpha_{b-}f(x) = \frac{1}{\Gamma(\alpha)}\int_x^b (t-x)^{\alpha-1}f(t)dt, \quad b > x,$$

where $\Gamma(\alpha) = \displaystyle\int_0^{+\infty} e^{-u}u^{\alpha-1}du$. Here $J^0_{a+}f(x) = J^0_{b-}f(x) = f(x)$.
In the case of $\alpha = 1$, the fractional integral reduces to the classical integral.

Due to the wide application of fractional integrals, some authors extended to study fractional Ostrowski type inequalities for functions of different classes, see [27] and the references cited therein. In [36], Raina introduced a class of functions defined formally by

$$\mathcal{F}^\sigma_{\rho,\lambda}(x) = \mathcal{F}^{\sigma(0),\sigma(1),\ldots}_{\rho,\lambda}(x) = \sum_{k=0}^{+\infty}\frac{\sigma(k)}{\Gamma(\rho k + \lambda)}x^k \quad (\rho, \lambda > 0; |x| < \mathbb{R}), \tag{1.2}$$

where the coefficients $(\sigma(k), k \in \mathbb{N} \cup \{0\})$ is a bounded sequence of positive real numbers. With the help of (1.2), Raina [36] and Agarwal et al. [2] defined the following left-sided and right-sided fractional integral operators respectively, as follows:

$$\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a+;w}\varphi\right)(x)=\int_a^x (x-t)^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[w(x-t)^{\rho}]\varphi(t)dt \quad (x>a>0), \tag{1.3}$$

$$\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b-;w}\varphi\right)(x)=\int_x^b (t-x)^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[w(t-x)^{\rho}]\varphi(t)dt \quad (0<x<b), \tag{1.4}$$

where $\lambda,\rho>0$, $w\in\mathbb{R}$ and $\varphi(t)$ is such that the integral on the right side exits. It is easy to verify that $\mathcal{J}^{\sigma}_{\rho,\lambda,a+;w}\varphi(x)$ and $\mathcal{J}^{\sigma}_{\rho,\lambda,b-;w}\varphi(x)$ are bounded integral operators on $L_1(a,b)$, if

$$\mathfrak{R}:=\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]<\infty.$$

In fact, for $\varphi\in L_1(a,b)$, we have

$$\|\mathcal{J}^{\sigma}_{\rho,\lambda,a+;w}\varphi(x)\|_1\le\mathfrak{R}(b-a)^{\lambda}\|\varphi\|_1$$

and

$$\|\mathcal{J}^{\sigma}_{\rho,\lambda,b-;w}\varphi(x)\|_1\le\mathfrak{R}(b-a)^{\lambda}\|\varphi\|_1,$$

where

$$\|\varphi\|_p:=\left(\int_a^b|\varphi(t)|^p dt\right)^{\frac{1}{p}}.$$

The importance of these operators stems indeed from their generality. Many useful fractional integral operators can be obtained by specializing the coefficient $\sigma(k)$. For instance the classical Riemann-Liouville fractional integrals J^{α}_{a+} and J^{α}_{b-} of order α follow easily by setting $\lambda=\alpha$, $\sigma(0)=1$ and $w=0$ in (1.3) and (1.4).

Next, let us evoke some definitions.

Definition 1.3. [16] A function $f:[0,+\infty)\longrightarrow\mathbb{R}$ is said to be s-convex in the second sense, if

$$f(\lambda x+(1-\lambda)y)\le\lambda^s f(x)+(1-\lambda)^s f(y) \tag{1.5}$$

for all $x,y\in[0,+\infty)$, $\lambda\in[0,1]$ and $s\in(0,1]$.

It is clear that a 1-convex function must be convex on $[0,+\infty)$ as usual. The s-convex functions in the second sense have been investigated in [16].

Definition 1.4. [7] A set $K\subseteq\mathbb{R}^n$ is said to be invex with respect to the mapping $\eta:K\times K\longrightarrow\mathbb{R}^n$, if $x+t\eta(y,x)\in K$ for every $x,y\in K$ and $t\in[0,1]$.

Notice that every convex set is invex with respect to the mapping $\eta(y,x)=y-x$, but the converse is not necessarily true, see [7],[42] and the references therein.

Definition 1.5. [31] The function f defined on the invex set $K\subseteq\mathbb{R}^n$ is said to be preinvex with respect η, if for every $x,y\in K$ and $t\in[0,1]$, we have that

$$f\left(x+t\eta(y,x)\right)\le(1-t)f(x)+tf(y).$$

The concept of preinvexity is more general than convexity since every convex function is preinvex with respect to the mapping $\eta(y,x) = y-x$, but the converse is not true.

The aim of this paper is to establish some generalizations of Ostrowski type inequalities using new identity given in Section 2 for generalized beta (r,g)-preinvex functions via generalized fractional integral operators. In Section 3, some applications to special means are obtain. In Section 4, some conclusions and future research are given. These results not only extend the results appeared in the literature [43] but also provide new estimates on these type.

2. Main results

Definition 2.1. [14] A set $K \subseteq \mathbb{R}^n$ is said to be m-invex with respect to the mapping $\eta : K\times K\times(0,1] \longrightarrow \mathbb{R}^n$ for some fixed $m \in (0,1]$, if $mx+t\eta(y,mx) \in K$ holds for each $x, y \in K$ and any $t \in [0,1]$.

Remark 2.2. In Definition 2.1, under certain conditions, the mapping $\eta(y,mx)$ could reduce to $\eta(y,x)$. For example when $m = 1$, then the m-invex set degenerates an invex set on K.

We next give new definition, to be referred as generalized beta (r,g)-preinvex function.

Definition 2.3. Let $K \subseteq \mathbb{R}$ be an open nonempty m-invex set with respect to $\eta : K \times K \times (0,1] \longrightarrow \mathbb{R}$, $g : [0,1] \longrightarrow [0,1]$ be a differentiable function and $\varphi : I \longrightarrow K$ is a continuous function. The function $f : K \longrightarrow (0,+\infty)$ is said to be generalized beta (r,g)-preinvex with respect to η, if

$$f\left(m\varphi(x) + g(t)\eta(\varphi(y),\varphi(x),m)\right) \leq M_r(f(\varphi(x)), f(\varphi(y)), m, p, q; g(t)) \tag{2.1}$$

holds for some fixed $m \in (0,1]$, for any fixed $p, q > -1$ and for all $x, y \in I, t \in [0,1]$, where

$$M_r(f(\varphi(x)), f(\varphi(y)), m, p, q; g(t))$$
$$:= \begin{cases} \left[mg^p(t)(1-g(t))^q f^r(\varphi(x)) + g^q(t)(1-g(t))^p f^r(\varphi(y))\right]^{\frac{1}{r}}, & r \neq 0; \\ \left[f(\varphi(x))\right]^{mg^p(t)(1-g(t))^q}\left[f(\varphi(y))\right]^{g^q(t)(1-g(t))^p}, & r = 0, \end{cases}$$

is the weighted power mean of order r for positive numbers $f(\varphi(x))$ and $f(\varphi(y))$.

Remark 2.4. In Definition 2.3, it is worthwhile to note that the class of generalized beta (r,g)-preinvex function is a generalization of the class of s-convex in the second sense function given in Definition 1.3. For $p = 0$, $q = s$ and $g(t) = t$, we get the notion of generalized $(r; s, m, \varphi)$-preinvex function [20]. Also, for $r = 1, p = 0, q = s$, $g(t) = t$ and $\varphi(x) = x$, $\forall x \in I$, we get the notion of generalized (s,m)-preinvex function [14].

Throughout this paper we denote $\alpha^* = \frac{x-m\varphi(a)}{\eta(\varphi(b),\varphi(a),m)}$ and

$$I_{f,g,\eta,\varphi}(x; \lambda, \rho, w, m, a, b)$$

$$
\begin{aligned}
&= g^{\lambda}(\alpha^*)\mathcal{F}^{\sigma}_{\rho,\lambda+1}\big[w\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(\alpha^*)\big]\frac{f(m\varphi(a)+g(\alpha^*)\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}\\
&-g^{\lambda}(0)\mathcal{F}^{\sigma}_{\rho,\lambda+1}\big[w\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(0)\big]\frac{f(m\varphi(a)+g(0)\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}\\
&+(1-g(1))^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}\big[w\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(1))^{\rho}\big]\frac{f(m\varphi(a)+g(1)\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}\\
&-(1-g(\alpha^*))^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}\big[w\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(\alpha^*))^{\rho}\big]\\
&\times\frac{f(m\varphi(a)+g(\alpha^*)\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}-\frac{\lambda}{\eta^{\lambda+1}(\varphi(b),\varphi(a),m)}\\
&\times\Bigg\{\int_{m\varphi(a)+g(0)\eta(\varphi(b),\varphi(a),m)}^{m\varphi(a)+g(\alpha^*)\eta(\varphi(b),\varphi(a),m)}(x-m\varphi(a))^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[w(x-m\varphi(a))^{\rho}]f(x)dx\\
&+\int_{m\varphi(a)+g(\alpha^*)\eta(\varphi(b),\varphi(a),m)}^{m\varphi(a)+g(1)\eta(\varphi(b),\varphi(a),m)}(m\varphi(a)+\eta(\varphi(b),\varphi(a),m)-x)^{\lambda-1}\\
&\times\mathcal{F}^{\sigma}_{\rho,\lambda}[w(m\varphi(a)+\eta(\varphi(b),\varphi(a),m)-x)^{\rho}]f(x)dx\Bigg\}. \qquad (2.2)
\end{aligned}
$$

In this section, in order to prove our main results regarding some generalizations of Ostrowski type inequalities for generalized beta (r,g)-preinvex functions via generalized fractional integral operators, we need the following new interesting lemma.

Lemma 2.5. *Let $\varphi: I\longrightarrow K$ be a continuous function and $g:[0,1]\longrightarrow[0,1]$ be a differentiable function. Suppose $K\subseteq\mathbb{R}$ be an open nonempty m-invex subset with respect to $\eta: K\times K\times(0,1]\longrightarrow\mathbb{R}$ for some fixed $m\in(0,1]$ and let $\eta(\varphi(b),\varphi(a),m)>0$. Assume that $f:K\longrightarrow\mathbb{R}$ is a differentiable function on K°. If $f'\in L_1[m\varphi(a),m\varphi(a)+\eta(\varphi(b),\varphi(a),m)]$, then we have the following identity involving generalized fractional integral operators:*

$$
I_{f,g,\eta,\varphi}(x;\lambda,\rho,w,m,a,b)=\int_0^1\theta_g(t)f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))d[g(t)] \qquad (2.3)
$$

for each $t\in[0,1]$, where $\lambda,\rho>0$, $w\in\mathbb{R}$ and

$$
\theta_g(t):=\begin{cases} g^{\lambda}(t)\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(t)], & t\in[0,\alpha^*);\\ (1-g(t))^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(t))^{\rho}], & t\in[\alpha^*,1].\end{cases}
$$

Proof. Integrating by parts, we get

$$
\begin{aligned}
&\int_0^1\theta_g(t)f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))d[g(t)]\\
&=\int_0^{\alpha^*}g^{\lambda}(t)\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(t)]f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))d[g(t)]\\
&+\int_{\alpha^*}^1(1-g(t))^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(t))^{\rho}]\\
&\times f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))d[g(t)]
\end{aligned}
$$

$$= t^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w\eta^{\rho}(\varphi(b),\varphi(a),m)t^{\rho}]\frac{f(m\varphi(a)+t\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}\Bigg|_{g(0)}^{g(\alpha^*)}$$

$$-\lambda\int_{g(0)}^{g(\alpha^*)} t^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[w\eta^{\rho}(\varphi(b),\varphi(a),m)t^{\rho}]\frac{f(m\varphi(a)+t\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}dt$$

$$+(1-t)^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w\eta^{\rho}(\varphi(b),\varphi(a),m)(1-t)^{\rho}]\frac{f(m\varphi(a)+t\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}\Bigg|_{g(\alpha^*)}^{g(1)}$$

$$-\lambda\int_{g(\alpha^*)}^{g(1)}(1-t)^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[w\eta^{\rho}(\varphi(b),\varphi(a),m)(1-t)^{\rho}]$$

$$\times\frac{f(m\varphi(a)+t\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}dt$$

$$= g^{\lambda}(\alpha^*)\mathcal{F}^{\sigma}_{\rho,\lambda+1}\left[w\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(\alpha^*)\right]\frac{f(m\varphi(a)+g(\alpha^*)\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}$$

$$-g^{\lambda}(0)\mathcal{F}^{\sigma}_{\rho,\lambda+1}\left[w\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(0)\right]\frac{f(m\varphi(a)+g(0)\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}$$

$$+(1-g(1))^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}\left[w\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(1))^{\rho}\right]\frac{f(m\varphi(a)+g(1)\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}$$

$$-(1-g(\alpha^*))^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}\left[w\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(\alpha^*))^{\rho}\right]$$

$$\times\frac{f(m\varphi(a)+g(\alpha^*)\eta(\varphi(b),\varphi(a),m))}{\eta(\varphi(b),\varphi(a),m)}-\frac{\lambda}{\eta^{\lambda+1}(\varphi(b),\varphi(a),m)}$$

$$\times\Bigg\{\int_{m\varphi(a)+g(0)\eta(\varphi(b),\varphi(a),m)}^{m\varphi(a)+g(\alpha^*)\eta(\varphi(b),\varphi(a),m)}(x-m\varphi(a))^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[w(x-m\varphi(a))^{\rho}]f(x)dx$$

$$+\int_{m\varphi(a)+g(\alpha^*)\eta(\varphi(b),\varphi(a),m)}^{m\varphi(a)+g(1)\eta(\varphi(b),\varphi(a),m)}(m\varphi(a)+\eta(\varphi(b),\varphi(a),m)-x)^{\lambda-1}$$

$$\times\mathcal{F}^{\sigma}_{\rho,\lambda}[w(m\varphi(a)+\eta(\varphi(b),\varphi(a),m)-x)^{\rho}]f(x)dx\Bigg\}.$$

This completes the proof of the lemma. □

Remark 2.6. Under the same conditions as in Lemma 2.5 for $g(t)=t$, we get

$$I_{f,\eta,\varphi}(x;\lambda,\rho,w,m,a,b)$$

$$=\Bigg[\frac{(x-m\varphi(a))^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(x-m\varphi(a))^{\rho}]}{\eta^{\lambda+1}(\varphi(b),\varphi(a),m)}$$

$$+\frac{(m\varphi(a)+\eta(\varphi(b),\varphi(a),m)-x)^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(m\varphi(a)+\eta(\varphi(b),\varphi(a),m)-x)^{\rho}]}{\eta^{\lambda+1}(\varphi(b),\varphi(a),m)}\Bigg]f(x)$$

$$-\frac{\lambda}{\eta^{\lambda+1}(\varphi(b),\varphi(a),m)}$$

$$\times\Big[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,x-;w}f\right)(m\varphi(a))+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,x+;w}f\right)(m\varphi(a)+\eta(\varphi(b),\varphi(a),m))\Big]$$

$$= \int_0^1 \theta(t) f'(m\varphi(a) + t\eta(\varphi(b), \varphi(a), m))dt, \tag{2.4}$$

where

$$\theta(t) := \begin{cases} t^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w\eta^{\rho}(\varphi(b), \varphi(a), m)t^{\rho}], & t \in [0, \alpha^*); \\ (1-t)^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w\eta^{\rho}(\varphi(b), \varphi(a), m)(1-t)^{\rho}], & t \in [\alpha^*, 1]. \end{cases}$$

By using Lemma 2.5, one can extend to the following results.

Theorem 2.7. *Let $\varphi : I \longrightarrow A$ be a continuous function and $g : [0, 1] \longrightarrow [0, 1]$ a differentiable function. Suppose $A \subseteq \mathbb{R}$ be an open nonempty m-invex subset with respect to $\eta : A \times A \times (0, 1] \longrightarrow \mathbb{R}$ for some fixed $m \in (0, 1]$, for any fixed $p, q > -1$ and let $\eta(\varphi(b), \varphi(a), m) > 0$. Assume that $f : A \longrightarrow (0, +\infty)$ is a differentiable function on A°. If $0 < r \leq 1$ and f' is generalized beta (r, g)-preinvex function on $[m\varphi(a), m\varphi(a) + \eta(\varphi(b), \varphi(a), m)]$, then the following inequality for generalized fractional integral operators holds:*

$$\begin{aligned}
\left|I_{f,g,\eta,\varphi}(x; \lambda, \rho, w, m, a, b)\right| &\leq \mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b), \varphi(a), m)] \\
&\times \Bigg[\Bigg\{ mf'(\varphi(a))^r B^r_{g(\alpha^*)}\left(\lambda + \rho k + \frac{p}{r} + 1, \frac{q}{r} + 1\right) \\
&+ f'(\varphi(b))^r B^r_{g(\alpha^*)}\left(\lambda + \rho k + \frac{q}{r} + 1, \frac{p}{r} + 1\right)\Bigg\}^{\frac{1}{r}} \\
&+ \Bigg\{ mf'(\varphi(a))^r \Bigg[B_{g(1)}\left(\frac{p}{r} + 1, \lambda + \rho k + \frac{q}{r} + 1\right) \\
&- B_{g(\alpha^*)}\left(\frac{p}{r} + 1, \lambda + \rho k + \frac{q}{r} + 1\right)\Bigg]^r \\
&+ f'(\varphi(b))^r \Bigg[B_{g(1)}\left(\frac{q}{r} + 1, \lambda + \rho k + \frac{p}{r} + 1\right) \\
&- B_{g(\alpha^*)}\left(\frac{q}{r} + 1, \lambda + \rho k + \frac{p}{r} + 1\right)\Bigg]^r\Bigg\}^{\frac{1}{r}}\Bigg],
\end{aligned} \tag{2.5}$$

where $\lambda, \rho > 0$, $w \in \mathbb{R}$, $k = 0, 1, 2, \ldots,$ and

$$B_{g(x)}(a, b) := \int_{g(0)}^{g(x)} t^{a-1}(1-t)^{b-1}dt.$$

Proof. Let $0 < r \leq 1$. From Lemma 2.5, generalized beta (r, g)-preinvexity of f', Minkowski inequality and properties of modulus, we have

$$\begin{aligned}
&\left|I_{f,g,\eta,\varphi}(x; \lambda, \rho, w, m, a, b)\right| \\
&\leq \int_0^{\alpha^*} g^{\lambda}(t)\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b), \varphi(a), m)g^{\rho}(t)] \\
&\times f'(m\varphi(a) + g(t)\eta(\varphi(b), \varphi(a), m))d[g(t)]
\end{aligned}$$

$$+\int_{\alpha^*}^{1}|1-g(t)|^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)|1-g(t)|^{\rho}]$$
$$\times f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))d[g(t)]$$
$$\leq\int_{0}^{\alpha^*}g^{\lambda}(t)\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(t)]$$
$$\times\left[mg^{p}(t)(1-g(t))^{q}f'(\varphi(a))^{r}+g^{q}(t)(1-g(t))^{p}f'(\varphi(b))^{r}\right]^{\frac{1}{r}}d[g(t)]$$
$$+\int_{\alpha^*}^{1}(1-g(t))^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(t))^{\rho}]$$
$$\times\left[mg^{p}(t)(1-g(t))^{q}f'(\varphi(a))^{r}+g^{q}(t)(1-g(t))^{p}f'(\varphi(b))^{r}\right]^{\frac{1}{r}}d[g(t)]$$
$$\leq\Bigg\{mf'(\varphi(a))^{r}\Bigg[\int_{0}^{\alpha^*}g^{\lambda+\frac{p}{r}}(t)(1-g(t))^{\frac{q}{r}}$$
$$\times\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(t)]d[g(t)]\Bigg]^{r}$$
$$+f'(\varphi(b))^{r}\left(\int_{0}^{\alpha^*}g^{\lambda+\frac{q}{r}}(t)(1-g(t))^{\frac{p}{r}}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(t)]d[g(t)]\right)^{r}\Bigg\}^{\frac{1}{r}}$$
$$+\Bigg\{mf'(\varphi(a))^{r}\Bigg[\int_{\alpha^*}^{1}g^{\frac{p}{r}}(t)(1-g(t))^{\lambda+\frac{q}{r}}$$
$$\times\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(t))^{\rho}]d[g(t)]\Bigg]^{r}$$
$$+f'(\varphi(b))^{r}\Bigg[\int_{\alpha^*}^{1}g^{\frac{q}{r}}(t)(1-g(t))^{\lambda+\frac{p}{r}}$$
$$\times\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(t))^{\rho}]d[g(t)]\Bigg]^{r}\Bigg\}^{\frac{1}{r}}$$
$$=\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)]$$
$$\times\Bigg[\Bigg\{mf'(\varphi(a))^{r}B^{r}_{g(\alpha^*)}\left(\lambda+\rho k+\frac{p}{r}+1,\frac{q}{r}+1\right)$$
$$+f'(\varphi(b))^{r}B^{r}_{g(\alpha^*)}\left(\lambda+\rho k+\frac{q}{r}+1,\frac{p}{r}+1\right)\Bigg\}^{\frac{1}{r}}$$
$$+\Bigg\{mf'(\varphi(a))^{r}\Bigg[B_{g(1)}\left(\frac{p}{r}+1,\lambda+\rho k+\frac{q}{r}+1\right)$$
$$-B_{g(\alpha^*)}\left(\frac{p}{r}+1,\lambda+\rho k+\frac{q}{r}+1\right)\Bigg]^{r}$$

$$+f'(\varphi(b))^r\Bigg[B_{g(1)}\left(\frac{q}{r}+1,\lambda+\rho k+\frac{p}{r}+1\right)$$

$$-B_{g(\alpha^*)}\left(\frac{q}{r}+1,\lambda+\rho k+\frac{p}{r}+1\right)\Bigg]^r\Bigg\}^{\frac{1}{r}}\Bigg].$$

So, the proof of this theorem is complete. □

Corollary 2.8. *Under the same conditions as in Theorem 2.7, if we choose* $p=0$, $m=q=1$, $\eta(\varphi(b),\varphi(a),m)=\varphi(b)-m\varphi(a)$, $g(t)=t$ *and* $\varphi(x)=x$, *we get*

$$\Bigg|\left[\frac{(x-a)^\lambda\mathcal{F}^\sigma_{\rho,\lambda+1}[w(x-a)^\rho]+(b-x)^\lambda\mathcal{F}^\sigma_{\rho,\lambda+1}[w(b-x)^\rho]}{(b-a)^{\lambda+1}}\right]f(x)$$

$$-\frac{\lambda}{(b-a)^{\lambda+1}}\Big[\left(\mathcal{J}^\sigma_{\rho,\lambda,x-;w}f\right)(a)+\left(\mathcal{J}^\sigma_{\rho,\lambda,x+;w}f\right)(b)\Big]\Bigg|$$

$$\leq\left\{f'(a)^r\left(\mathcal{F}^{\sigma_1^*}_{\rho,\lambda+1}[|w|(b-a)^\rho]\right)^r+f'(b)^r\left(\mathcal{F}^{\sigma_2^*}_{\rho,\lambda+1}[|w|(x-a)^\rho]\right)^r\right\}^{\frac{1}{r}}$$

$$+\left\{f'(a)^r\left(\mathcal{F}^{\sigma_3^*}_{\rho,\lambda+1}[|w|(b-x)^\rho]\right)^r+f'(b)^r\left(\mathcal{F}^{\sigma_4^*}_{\rho,\lambda+1}[|w|(b-a)^\rho]\right)^r\right\}^{\frac{1}{r}},\tag{2.6}$$

where

$$\sigma_1^*(k)=\sigma(k)\beta\left(\frac{x-a}{b-a};\lambda+\rho k+1,\frac{1}{r}+1\right);$$

$$\sigma_2^*(k)=\sigma(k)\left(\frac{x-a}{b-a}\right)^{\lambda+\frac{1}{r}+1}\frac{1}{\lambda+\rho k+\frac{1}{r}+1};$$

$$\sigma_3^*(k)=\sigma(k)\left(\frac{b-x}{b-a}\right)^{\lambda+\frac{1}{r}+1}\frac{1}{\lambda+\rho k+\frac{1}{r}+1};$$

$$\sigma_4^*(k)=\sigma(k)\beta\left(\frac{b-x}{b-a};\lambda+\rho k+1,\frac{1}{r}+1\right).$$

Corollary 2.9. *If we choose* $r=\sigma(0)=1, w=0$ *in Corollary 2.8, the inequality (2.6) reduces to inequality (2.1) of ([43], Theorem 2.1).*

Theorem 2.10. *Let* $\varphi:I\longrightarrow A$ *be a continuous function and* $g:[0,1]\longrightarrow[0,1]$ *a differentiable function. Suppose* $A\subseteq\mathbb{R}$ *be an open nonempty* m*-invex subset with respect to* $\eta:A\times A\times(0,1]\longrightarrow\mathbb{R}$ *for some fixed* $m\in(0,1]$, *for any fixed* $\gamma,s>-1$ *and let* $\eta(\varphi(b),\varphi(a),m)>0$. *Assume that* $f:A\longrightarrow(0,+\infty)$ *is a differentiable function on* A°. *If* $0<r\leq 1$ *and* f'^q *is generalized beta* (r,g)*-preinvex function on* $[m\varphi(a),m\varphi(a)+\eta(\varphi(b),\varphi(a),m)]$, $q>1$, $p^{-1}+q^{-1}=1$, *then the following inequality for generalized fractional integral operators holds:*

$$\left|I_{f,g,\eta,\varphi}(x;\lambda,\rho,w,m,a,b)\right|\leq\mathcal{F}^{\sigma^*}_{\rho,\lambda+1}[|w|\eta^\rho(\varphi(b),\varphi(a),m)]$$

$$\times\Bigg\{ \left(g^{(\lambda+\rho k)p+1}(\alpha^*) - g^{(\lambda+\rho k)p+1}(0)\right)^{\frac{1}{p}}$$

$$\times\left[mf'(\varphi(a))^{rq}B^r_{g(\alpha^*)}\left(\frac{\gamma}{r}+1,\frac{s}{r}+1\right)+f'(\varphi(b))^{rq}B^r_{g(\alpha^*)}\left(\frac{s}{r}+1,\frac{\gamma}{r}+1\right)\right]^{\frac{1}{rq}}$$

$$+\left((1-g(\alpha^*))^{(\lambda+\rho k)p+1}-(1-g(1))^{(\lambda+\rho k)p+1}\right)^{\frac{1}{p}}$$

$$\times\Bigg[mf'(\varphi(a))^{rq}\left(B_{g(1)}\left(\frac{\gamma}{r}+1,\frac{s}{r}+1\right)-B_{g(\alpha^*)}\left(\frac{\gamma}{r}+1,\frac{s}{r}+1\right)\right)^r$$

$$+f'(\varphi(b))^{rq}\left(B_{g(1)}\left(\frac{s}{r}+1,\frac{\gamma}{r}+1\right)-B_{g(\alpha^*)}\left(\frac{s}{r}+1,\frac{\gamma}{r}+1\right)\right)^r\Bigg]^{\frac{1}{rq}}\Bigg\}, \tag{2.7}$$

where $\lambda,\rho>0$, $w\in\mathbb{R}$, $k=0,1,2,\dots,$ *and*

$$\sigma^*(k)=\sigma(k)\left(\frac{1}{(\lambda+\rho k)p+1}\right)^{\frac{1}{p}}.$$

Proof. Suppose that $q>1$ and $0<r\le 1$. From Lemma 2.5, generalized beta (r,g)-preinvexity of f'^q, Hölder inequality, Minkowski inequality and properties of modulus, we have

$$\left|I_{f,g,\eta,\varphi}(x;\lambda,\rho,w,m,a,b)\right|$$

$$\le\int_0^{\alpha^*}g^\lambda(t)\mathcal{F}^\sigma_{\rho,\lambda+1}[|w|\eta^\rho(\varphi(b),\varphi(a),m)g^\rho(t)]f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))d[g(t)]$$

$$+\int_{\alpha^*}^1|1-g(t)|^\lambda\mathcal{F}^\sigma_{\rho,\lambda+1}[|w|\eta^\rho(\varphi(b),\varphi(a),m)|1-g(t)|^\rho]$$

$$\times f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))d[g(t)]$$

$$\le\sum_{k=0}^{+\infty}\frac{\sigma(k)|w|^k\eta^{\rho k}(\varphi(b),\varphi(a),m)}{\Gamma(\lambda+\rho k+1)}\times\Bigg\{\left(\int_0^{\alpha^*}g^{(\lambda+\rho k)p}(t)d[g(t)]\right)^{\frac{1}{p}}$$

$$\times\left(\int_0^{\alpha^*}\left(f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))\right)^q d[g(t)]\right)^{\frac{1}{q}}$$

$$+\left(\int_{\alpha^*}^1(1-g(t))^{(\lambda+\rho k)p}dt\right)^{\frac{1}{p}}$$

$$\times\left(\int_{\alpha^*}^1\left(f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))\right)^q d[g(t)]\right)^{\frac{1}{q}}\Bigg\}$$

$$\le\sum_{k=0}^{+\infty}\frac{\sigma(k)|w|^k\eta^{\rho k}(\varphi(b),\varphi(a),m)}{\Gamma(\lambda+\rho k+1)}\times\Bigg\{\left(\int_0^{\alpha^*}g^{(\lambda+\rho k)p}(t)d[g(t)]\right)^{\frac{1}{p}}$$

$$\times\left(\int_0^{\alpha^*}\left[mg^\gamma(t)(1-g(t))^sf'(\varphi(a))^{rq}+g^s(t)(1-g(t))^\gamma f'(\varphi(b))^{rq}\right]^{\frac{1}{r}}d[g(t)]\right)^{\frac{1}{q}}$$

$$+\left(\int_{\alpha^*}^{1}(1-g(t))^{(\lambda+\rho k)p}d[g(t)]\right)^{\frac{1}{p}}$$

$$\times\left(\int_{\alpha^*}^{1}\left[mg^{\gamma}(t)(1-g(t))^{s}f'(\varphi(a))^{rq}+g^{s}(t)(1-g(t))^{\gamma}f'(\varphi(b))^{rq}\right]^{\frac{1}{r}}d[g(t)]\right)^{\frac{1}{q}}\Bigg\}$$

$$\leq\sum_{k=0}^{+\infty}\frac{\sigma(k)|w|^{k}\eta^{\rho k}(\varphi(b),\varphi(a),m)}{\Gamma(\lambda+\rho k+1)}\times\Bigg\{\left(\int_{0}^{\alpha^*}g^{(\lambda+\rho k)p}(t)d[g(t)]\right)^{\frac{1}{p}}$$

$$\times\Bigg[mf'(\varphi(a))^{rq}\left(\int_{0}^{\alpha^*}g^{\frac{\gamma}{r}}(t)(1-g(t))^{\frac{s}{r}}d[g(t)]\right)^{r}$$

$$+f'(\varphi(b))^{rq}\left(\int_{0}^{\alpha^*}g^{\frac{s}{r}}(t)(1-g(t))^{\frac{\gamma}{r}}d[g(t)]\right)^{r}\Bigg]^{\frac{1}{rq}}$$

$$+\left(\int_{\alpha^*}^{1}(1-g(t))^{(\lambda+\rho k)p}d[g(t)]\right)^{\frac{1}{p}}$$

$$\times\Bigg[mf'(\varphi(a))^{rq}\left(\int_{\alpha^*}^{1}g^{\frac{\gamma}{r}}(t)(1-g(t))^{\frac{s}{r}}d[g(t)]\right)^{r}$$

$$+f'(\varphi(b))^{rq}\left(\int_{\alpha^*}^{1}g^{\frac{s}{r}}(t)(1-g(t))^{\frac{\gamma}{r}}d[g(t)]\right)^{r}\Bigg]^{\frac{1}{rq}}\Bigg\}$$

$$=\mathcal{F}^{\sigma^*}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)]$$

$$\times\Bigg\{\left(g^{(\lambda+\rho k)p+1}(\alpha^*)-g^{(\lambda+\rho k)p+1}(0)\right)^{\frac{1}{p}}$$

$$\times\left[mf'(\varphi(a))^{rq}B^{r}_{g(\alpha^*)}\left(\frac{\gamma}{r}+1,\frac{s}{r}+1\right)+f'(\varphi(b))^{rq}B^{r}_{g(\alpha^*)}\left(\frac{s}{r}+1,\frac{\gamma}{r}+1\right)\right]^{\frac{1}{rq}}$$

$$+\left((1-g(\alpha^*))^{(\lambda+\rho k)p+1}-(1-g(1))^{(\lambda+\rho k)p+1}\right)^{\frac{1}{p}}$$

$$\times\Bigg[mf'(\varphi(a))^{rq}\left(B_{g(1)}\left(\frac{\gamma}{r}+1,\frac{s}{r}+1\right)-B_{g(\alpha^*)}\left(\frac{\gamma}{r}+1,\frac{s}{r}+1\right)\right)^{r}$$

$$+f'(\varphi(b))^{rq}\left(B_{g(1)}\left(\frac{s}{r}+1,\frac{\gamma}{r}+1\right)-B_{g(\alpha^*)}\left(\frac{s}{r}+1,\frac{\gamma}{r}+1\right)\right)^{r}\Bigg]^{\frac{1}{rq}}\Bigg\}.$$

So, the proof of this theorem is complete. □

Corollary 2.11. *Under the same conditions as in Theorem 2.10, if we choose $\gamma = 0$, $m = s = 1$, $\eta(\varphi(b), \varphi(a), m) = \varphi(b) - m\varphi(a)$, $g(t) = t$ and $\varphi(x) = x$, we get*

$$\begin{aligned}
&\Bigg|\left[\frac{(x-a)^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(x-a)^{\rho}] + (b-x)^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-x)^{\rho}]}{(b-a)^{\lambda+1}}\right]f(x) \\
&\quad - \frac{\lambda}{(b-a)^{\lambda+1}}\Big[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,x-;w}f\right)(a) + \left(\mathcal{J}^{\sigma}_{\rho,\lambda,x+;w}f\right)(b)\Big]\Bigg| \\
&\leq \left(\frac{r}{r+1}\right)^{\frac{1}{q}} \frac{1}{(b-a)^{\lambda+\frac{2}{rq}+\frac{1}{p}}} \\
&\times\Bigg\{\Big[\left[(b-a)^{\frac{1}{r}+1} - (b-x)^{\frac{1}{r}+1}\right]^{r} f'(a)^{rq} + (x-a)^{2} f'(b)^{rq}\Big]^{\frac{1}{rq}} \\
&\times (x-a)^{\lambda+\frac{1}{p}}\mathcal{F}^{\sigma^{*}}_{\rho,\lambda+1}[|w|(x-a)^{\rho}] \\
&+\Big[(b-x)^{2} f'(a)^{rq} + \left[(b-a)^{\frac{1}{r}+1} - (x-a)^{\frac{1}{r}+1}\right]^{r} f'(b)^{rq}\Big]^{\frac{1}{rq}} \\
&\times (b-x)^{\lambda+\frac{1}{p}}\mathcal{F}^{\sigma^{*}}_{\rho,\lambda+1}[|w|(b-x)^{\rho}]\Bigg\}. \qquad (2.8)
\end{aligned}$$

Theorem 2.12. *Let $\varphi : I \longrightarrow A$ be a continuous function and $g : [0,1] \longrightarrow [0,1]$ a differentiable function. Suppose $A \subseteq \mathbb{R}$ be an open nonempty m-invex subset with respect to $\eta : A \times A \times (0,1] \longrightarrow \mathbb{R}$ for any fixed $m \in (0,1]$, for any fixed $\gamma, s > -1$ and let $\eta(\varphi(b), \varphi(a), m) > 0$. Assume that $f : A \longrightarrow (0,+\infty)$ is a differentiable function on A°. If $0 < r \leq 1$ and f'^{q} is generalized beta (r,g)-preinvex function on $[m\varphi(a), m\varphi(a) + \eta(\varphi(b), \varphi(a), m)]$, $q \geq 1$, then the following inequality for generalized fractional integral operators holds:*

$$\begin{aligned}
&\left|I_{f,g,\eta,\varphi}(x; \lambda, \rho, w, m, a, b)\right| \\
&\leq \mathcal{F}^{\sigma^{**}}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b), \varphi(a), m)]\Bigg\{\left[g^{\rho k+1}(\alpha^{*}) - g^{\rho k+1}(0)\right]^{1-\frac{1}{q}} \\
&\times\Bigg[m f'(\varphi(a))^{rq} B^{r}_{g(\alpha^{*})}\left(\lambda q + \rho k + \frac{\gamma}{r} + 1, \frac{s}{r} + 1\right) \\
&+ f'(\varphi(b))^{rq} B^{r}_{g(\alpha^{*})}\left(\lambda q + \rho k + \frac{s}{r} + 1, \frac{\gamma}{r} + 1\right)\Bigg]^{\frac{1}{rq}} \\
&+\Big[(1-g(\alpha^{*}))^{\rho k+1} - (1-g(1))^{\rho k+1}\Big]^{1-\frac{1}{q}} \\
&+\Bigg[m f'(\varphi(a))^{rq}\Bigg[B_{g(1)}\left(\frac{\gamma}{r} + 1, \lambda q + \rho k + \frac{s}{r} + 1\right) \\
&- B_{g(\alpha^{*})}\left(\frac{\gamma}{r} + 1, \lambda q + \rho k + \frac{s}{r} + 1\right)\Bigg]^{r}
\end{aligned}$$

$$+f'(\varphi(b))^{rq}\Bigg[B_{g(1)}\left(\frac{s}{r}+1,\lambda q+\rho k+\frac{\gamma}{r}+1\right)$$

$$-B_{g(\alpha^*)}\left(\frac{s}{r}+1,\lambda q+\rho k+\frac{\gamma}{r}+1\right)\Bigg]^r\Bigg]^{\frac{1}{rq}}\Bigg\}, \tag{2.9}$$

where $\lambda,\rho>0$, $w\in\mathbb{R}$, $k=0,1,2,\dots,$ *and*

$$\sigma^{**}(k)=\sigma(k)\left(\frac{1}{\rho k+1}\right)^{1-\frac{1}{q}}.$$

Proof. Suppose that $q\geq 1$ and $0<r\leq 1$. From Lemma 2.5, generalized beta (r,g)-preinvexity of f'^q, the well-known power mean inequality, Minkowski inequality and properties of modulus, we have

$$\begin{aligned}
&\left|I_{f,g,\eta,\varphi}(x;\lambda,\rho,w,m,a,b)\right|\\
&\leq\int_0^{\alpha^*}g^{\lambda}(t)\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(t)]\\
&\quad\times f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))d[g(t)]\\
&+\int_{\alpha^*}^1|1-g(t)|^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)|1-g(t)|^{\rho}]\\
&\quad\times f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))d[g(t)]\\
&\leq\left(\int_0^{\alpha^*}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(t)]d[g(t)]\right)^{1-\frac{1}{q}}\\
&\quad\times\Bigg[\int_0^{\alpha^*}g^{\lambda q}(t)\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)g^{\rho}(t)]\\
&\quad\times\left(f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))\right)^q d[g(t)]\Bigg]^{\frac{1}{q}}\\
&+\left(\int_{\alpha^*}^1\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(t))^{\rho}]d[g(t)]\right)^{1-\frac{1}{q}}\\
&\times\Bigg[\int_{\alpha^*}^1(1-g(t))^{\lambda q}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)(1-g(t))^{\rho}]\\
&\quad\times\left(f'(m\varphi(a)+g(t)\eta(\varphi(b),\varphi(a),m))\right)^q d[g(t)]\Bigg]^{\frac{1}{q}}\\
&\leq\left(\sum_{k=0}^{+\infty}\frac{\sigma(k)|w|^k\eta^{\rho k}(\varphi(b),\varphi(a),m)}{\Gamma(\lambda+\rho k+1)}\int_0^{\alpha^*}g^{\rho k}(t)d[g(t)]\right)^{1-\frac{1}{q}}\\
&\quad\times\Bigg[\sum_{k=0}^{+\infty}\frac{\sigma(k)|w|^k\eta^{\rho k}(\varphi(b),\varphi(a),m)}{\Gamma(\lambda+\rho k+1)}
\end{aligned}$$

$$\times \int_0^{\alpha^*} g^{\lambda q+\rho k}(t)\Big[mg^{\gamma}(t)(1-g(t))^s f'(\varphi(a))^{rq}$$

$$+g^s(t)(1-g(t))^{\gamma} f'(\varphi(b))^{rq}\Big]^{\frac{1}{r}} d[g(t)]\Bigg]^{\frac{1}{q}}$$

$$+\left(\sum_{k=0}^{+\infty}\frac{\sigma(k)|w|^k\eta^{\rho k}(\varphi(b),\varphi(a),m)}{\Gamma(\lambda+\rho k+1)}\int_{\alpha^*}^{1}(1-g(t))^{\rho k}d[g(t)]\right)^{1-\frac{1}{q}}$$

$$\times\Bigg[\sum_{k=0}^{+\infty}\frac{\sigma(k)|w|^k\eta^{\rho k}(\varphi(b),\varphi(a),m)}{\Gamma(\lambda+\rho k+1)}$$

$$\times\int_{\alpha^*}^{1}(1-g(t))^{\lambda q+\rho k}\Big[mg^{\gamma}(t)(1-g(t))^s f'(\varphi(a))^{rq}$$

$$\times+g^s(t)(1-g(t))^{\gamma} f'(\varphi(b))^{rq}\Big]^{\frac{1}{r}} d[g(t)]\Bigg]^{\frac{1}{q}}$$

$$\leq \mathcal{F}^{\sigma^{**}}_{\rho,\lambda+1}[|w|\eta^{\rho}(\varphi(b),\varphi(a),m)]\Bigg\{\Big[g^{\rho k+1}(\alpha^*)-g^{\rho k+1}(0)\Big]^{1-\frac{1}{q}}$$

$$\times\Bigg[mf'(\varphi(a))^{rq}B^r_{g(\alpha^*)}\left(\lambda q+\rho k+\frac{\gamma}{r}+1,\frac{s}{r}+1\right)$$

$$+f'(\varphi(b))^{rq}B^r_{g(\alpha^*)}\left(\lambda q+\rho k+\frac{s}{r}+1,\frac{\gamma}{r}+1\right)\Bigg]^{\frac{1}{rq}}$$

$$+\Big[(1-g(\alpha^*))^{\rho k+1}-(1-g(1))^{\rho k+1}\Big]^{1-\frac{1}{q}}$$

$$+\Bigg[mf'(\varphi(a))^{rq}\Bigg[B_{g(1)}\left(\frac{\gamma}{r}+1,\lambda q+\rho k+\frac{s}{r}+1\right)$$

$$-B_{g(\alpha^*)}\left(\frac{\gamma}{r}+1,\lambda q+\rho k+\frac{s}{r}+1\right)\Bigg]^r$$

$$+f'(\varphi(b))^{rq}\Bigg[B_{g(1)}\left(\frac{s}{r}+1,\lambda q+\rho k+\frac{\gamma}{r}+1\right)$$

$$-B_{g(\alpha^*)}\left(\frac{s}{r}+1,\lambda q+\rho k+\frac{\gamma}{r}+1\right)\Bigg]^r\Bigg]^{\frac{1}{rq}}\Bigg\}.$$

So, the proof of this theorem is complete. $\square$

Corollary 2.13. *Under the same conditions as in Theorem 2.12, if we choose $\gamma = 0$, $r = m = s = 1$, $\eta(\varphi(b), \varphi(a), m) = \varphi(b) - m\varphi(a)$, $g(t) = t$ and $\varphi(x) = x$, we get*

$$\left| \left[\frac{(x-a)^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(x-a)^{\rho}] + (b-x)^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-x)^{\rho}]}{(b-a)^{\lambda+1}} \right] f(x) \right.$$

$$\left. - \frac{\lambda}{(b-a)^{\lambda+1}} \left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,x-;w} f\right)(a) + \left(\mathcal{J}^{\sigma}_{\rho,\lambda,x+;w} f\right)(b) \right] \right|$$

$$\leq \left(\mathcal{F}^{\sigma_1^*}_{\rho,\lambda+1}[|w|(x-a)^{\rho}] \right)^{1-\frac{1}{q}}$$

$$\times \left[f'(a)^q \mathcal{F}^{\sigma_2^*}_{\rho,\lambda+1}[|w|(b-a)^{\rho}] + f'(b)^q \mathcal{F}^{\sigma_3^*}_{\rho,\lambda+1}[|w|(x-a)^{\rho}] \right]^{\frac{1}{q}}$$

$$+ \left(\mathcal{F}^{\sigma_4^*}_{\rho,\lambda+1}[|w|(b-x)^{\rho}] \right)^{1-\frac{1}{q}}$$

$$\times \left[f'(a)^q \mathcal{F}^{\sigma_5^*}_{\rho,\lambda+1}[|w|(b-x)^{\rho}] + f'(b)^q \mathcal{F}^{\sigma_6^*}_{\rho,\lambda+1}[|w|(b-a)^{\rho}] \right]^{\frac{1}{q}}, \tag{2.10}$$

where

$$\sigma_1^*(k) = \sigma(k) \left(\frac{x-a}{b-a} \right) \frac{1}{\rho k + 1};$$

$$\sigma_2^*(k) = \sigma(k) \beta \left(\frac{x-a}{b-a}; \lambda q + \rho k + 1, 2 \right);$$

$$\sigma_3^*(k) = \sigma(k) \left(\frac{x-a}{b-a} \right)^{\lambda q + 2} \frac{1}{\lambda q + \rho k + 2};$$

$$\sigma_4^*(k) = \sigma(k) \left(\frac{b-x}{b-a} \right) \frac{1}{\rho k + 1};$$

$$\sigma_5^*(k) = \sigma(k) \left(\frac{b-x}{b-a} \right)^{\lambda q + 2} \frac{1}{\lambda q + \rho k + 2};$$

$$\sigma_6^*(k) = \sigma(k) \beta \left(\frac{b-x}{b-a}; \lambda q + \rho k + 1, 2 \right).$$

Corollary 2.14. *If we choose $\sigma(0) = 1, w = 0$ in Corollary 2.13, the inequality (2.10) reduces to inequality (2.4) of ([43], Theorem 2.3). Also, under the same conditions as in Theorem 2.12 for $q = 1$, we get Theorem 2.7.*

3. Applications to special means

Definition 3.1. [9] A function $M : \mathbb{R}_+^2 \longrightarrow \mathbb{R}_+$, is called a Mean function if it has the following properties:

(1) Homogeneity: $M(ax, ay) = aM(x, y)$, for all $a > 0$,
(2) Symmetry: $M(x, y) = M(y, x)$,
(3) Reflexivity: $M(x, x) = x$,
(4) Monotonicity: If $x \leq x'$ and $y \leq y'$, then $M(x, y) \leq M(x', y')$,
(5) Internality: $\min\{x, y\} \leq M(x, y) \leq \max\{x, y\}$.

We consider some means for arbitrary positive real numbers α, β $(\alpha \neq \beta)$.

(1) The arithmetic mean:
$$A := A(\alpha, \beta) = \frac{\alpha + \beta}{2}$$
(2) The geometric mean:
$$G := G(\alpha, \beta) = \sqrt{\alpha\beta}$$
(3) The harmonic mean:
$$H := H(\alpha, \beta) = \frac{2}{\frac{1}{\alpha} + \frac{1}{\beta}}$$
(4) The power mean:
$$P_r := P_r(\alpha, \beta) = \left(\frac{\alpha^r + \beta^r}{2}\right)^{\frac{1}{r}}, \quad r \geq 1.$$
(5) The identric mean:
$$I := I(\alpha, \beta) = \begin{cases} \frac{1}{e}\left(\frac{\beta^\beta}{\alpha^\alpha}\right), & \alpha \neq \beta; \\ \alpha, & \alpha = \beta. \end{cases}$$
(6) The logarithmic mean:
$$L := L(\alpha, \beta) = \frac{\beta - \alpha}{\ln(\beta) - \ln(\alpha)}.$$
(7) The generalized log-mean:
$$L_p := L_p(\alpha, \beta) = \left[\frac{\beta^{p+1} - \alpha^{p+1}}{(p+1)(\beta - \alpha)}\right]^{\frac{1}{p}}; \quad p \in \mathbb{R} \setminus \{-1, 0\}.$$
(8) The weighted p-power mean:
$$M_p\begin{pmatrix} \alpha_1, & \alpha_2, & \cdots & , \alpha_n \\ u_1, & u_2, & \cdots & , u_n \end{pmatrix} = \left(\sum_{i=1}^{n} \alpha_i u_i^p\right)^{\frac{1}{p}}$$
where $0 \leq \alpha_i \leq 1$, $u_i > 0$ $(i = 1, 2, \ldots, n)$ with $\sum_{i=1}^{n} \alpha_i = 1$.

It is well known that L_p is monotonic nondecreasing over $p \in \mathbb{R}$ with $L_{-1} := L$ and $L_0 := I$. In particular, we have the following inequality $H \leq G \leq L \leq I \leq A$. Now, let a and b be positive real numbers such that $a < b$. Consider the function $M := M(\varphi(a), \varphi(b)) : [\varphi(a), \varphi(a)+\eta(\varphi(b), \varphi(a))] \times [\varphi(a), \varphi(a)+\eta(\varphi(b), \varphi(a))] \longrightarrow \mathbb{R}_+$, which is one of the above mentioned means and $\varphi : I \longrightarrow A$ be a continuous function and $g : [0,1] \longrightarrow [0,1]$ a differentiable function. Therefore one can obtain various inequalities using the results of Section 2 for these means as follows: By replacing $\eta(\varphi(y), \varphi(x), m)$ with $\eta(\varphi(y), \varphi(x))$ and setting $\eta(\varphi(a), \varphi(b)) = M(\varphi(a), \varphi(b))$ for value $m = 1$ in (2.5), (2.7) and (2.9), one can obtain the following interesting inequalities involving means:

$$
\begin{aligned}
&\left|I_{f,g,M(\cdot,\cdot),\varphi}(x;\lambda,\rho,w,1,a,b)\right| \leq \mathcal{F}^{\sigma}_{\rho,\lambda+1}[|w|M^{\rho}(\varphi(a),\varphi(b))] \\
&\times\Bigg[\Bigg\{ f'(\varphi(a))^r B^r_{g(\alpha_1^*)}\left(\lambda+\rho k+\frac{p}{r}+1, \frac{q}{r}+1\right) \\
&+ f'(\varphi(b))^r B^r_{g(\alpha_1^*)}\left(\lambda+\rho k+\frac{q}{r}+1, \frac{p}{r}+1\right)\Bigg\}^{\frac{1}{r}} \\
&+\Bigg\{ f'(\varphi(a))^r \left(B_{g(1)}\left(\frac{p}{r}+1, \lambda+\rho k+\frac{q}{r}+1\right) - B_{g(\alpha_1^*)}\left(\frac{p}{r}+1, \lambda+\rho k+\frac{q}{r}+1\right)\right)^r \\
&+ f'(\varphi(b))^r \Bigg[B_{g(1)}\left(\frac{q}{r}+1, \lambda+\rho k+\frac{p}{r}+1\right) \\
&- B_{g(\alpha_1^*)}\left(\frac{q}{r}+1, \lambda+\rho k+\frac{p}{r}+1\right)\Bigg]^r\Bigg\}^{\frac{1}{r}}\Bigg], \qquad (3.1)
\end{aligned}
$$

where

$$
\alpha_1^* = \frac{x-\varphi(a)}{M(\varphi(a),\varphi(b))};
$$

$$
\begin{aligned}
&\left|I_{f,g,M(\cdot,\cdot),\varphi}(x;\lambda,\rho,w,1,a,b)\right| \leq \mathcal{F}^{\sigma^*}_{\rho,\lambda+1}[|w|M^{\rho}(\varphi(a),\varphi(b))] \\
&\times\Bigg\{ \left(g^{(\lambda+\rho k)p+1}(\alpha_1^*) - g^{(\lambda+\rho k)p+1}(0)\right)^{\frac{1}{p}} \\
&\times\left[f'(\varphi(a))^{rq} B^r_{g(\alpha_1^*)}\left(\frac{\gamma}{r}+1, \frac{s}{r}+1\right) + f'(\varphi(b))^{rq} B^r_{g(\alpha_1^*)}\left(\frac{s}{r}+1, \frac{\gamma}{r}+1\right)\right]^{\frac{1}{rq}} \\
&+ \left((1-g(\alpha_1^*))^{(\lambda+\rho k)p+1} - (1-g(1))^{(\lambda+\rho k)p+1}\right)^{\frac{1}{p}} \\
&\times\Bigg[f'(\varphi(a))^{rq}\left(B_{g(1)}\left(\frac{\gamma}{r}+1, \frac{s}{r}+1\right) - B_{g(\alpha_1^*)}\left(\frac{\gamma}{r}+1, \frac{s}{r}+1\right)\right)^r \\
&+ f'(\varphi(b))^{rq}\left(B_{g(1)}\left(\frac{s}{r}+1, \frac{\gamma}{r}+1\right) - B_{g(\alpha_1^*)}\left(\frac{s}{r}+1, \frac{\gamma}{r}+1\right)\right)^r\Bigg]^{\frac{1}{rq}}\Bigg\}, \qquad (3.2)
\end{aligned}
$$

$$
\left|I_{f,g,M(\cdot,\cdot),\varphi}(x;\lambda,\rho,w,1,a,b)\right|
$$

$$
\begin{aligned}
&\leq \mathcal{F}^{\sigma^{**}}_{\rho,\lambda+1}[|w|M^{\rho}(\varphi(a),\varphi(b))]\Bigg\{\left[g^{\rho k+1}(\alpha_1^*)-g^{\rho k+1}(0)\right]^{1-\frac{1}{q}}\\
&\times\Bigg[f'(\varphi(a))^{rq}B^{r}_{g(\alpha_1^*)}\left(\lambda q+\rho k+\frac{\gamma}{r}+1,\frac{s}{r}+1\right)\\
&+f'(\varphi(b))^{rq}B^{r}_{g(\alpha_1^*)}\left(\lambda q+\rho k+\frac{s}{r}+1,\frac{\gamma}{r}+1\right)\Bigg]^{\frac{1}{rq}}\\
&+\left[(1-g(\alpha_1^*))^{\rho k+1}-(1-g(1))^{\rho k+1}\right]^{1-\frac{1}{q}}\\
&+\Bigg[f'(\varphi(a))^{rq}\Bigg[B_{g(1)}\left(\frac{\gamma}{r}+1,\lambda q+\rho k+\frac{s}{r}+1\right)\\
&-B_{g(\alpha_1^*)}\left(\frac{\gamma}{r}+1,\lambda q+\rho k+\frac{s}{r}+1\right)\Bigg]^{r}\\
&+f'(\varphi(b))^{rq}\Bigg[B_{g(1)}\left(\frac{s}{r}+1,\lambda q+\rho k+\frac{\gamma}{r}+1\right)\\
&-B_{g(\alpha_1^*)}\left(\frac{s}{r}+1,\lambda q+\rho k+\frac{\gamma}{r}+1\right)\Bigg]^{r}\Bigg]^{\frac{1}{rq}}\Bigg\}. \qquad (3.3)
\end{aligned}
$$

Letting $M(\varphi(a),\varphi(b)) := A, G, H, P_r, I, L, L_p, M_p$ in (3.1), (3.2) and (3.3), we get the inequalities involving means for a particular choice of a differentiable generalized beta (r,g)-preinvex functions f. The details are left to the interested reader.

4. Conclusion

In the present paper, the notion of generalized beta (r,g)-preinvex function was applied for established some new generalizations of Ostrowski type inequalities via fractional integral operators. These results not only extend the results appeared in the literature [43] but also provide new estimates on these type. At the end, some applications to special means are obtained. Motivated by this new interesting class of generalized beta (r,g)-preinvex functions we can indeed see to be vital for fellow researchers and scientists working in the same domain. We conclude that our methods considered here may be a stimulant for further investigations concerning Hermite-Hadamard, Ostrowski and Simpson type integral inequalities for various kinds of preinvex functions involving classical integrals, Riemann-Liouville fractional integrals, k-fractional integrals, local fractional integrals, fractional integral operators, q-calculus, (p,q)-calculus, time scale calculus and conformable fractional integrals.

Acknowledgement. We thank anonymous referee for their valuable suggestions regarding the manuscript.

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[1] Department of Mathematics, Faculty of Technical Science, University "Ismail Qemali", Vlora, Albania.
E-mail address: artionkashuri@gmail.com; rozanaliko86@gmail.com

[2] College of Science, China Three Gorges University, 443002, Yichang, P. R. China.
E-mail address: tingsongdu@ctgu.edu.cn