Abstract. Let $R$ be a commutative ring with identity, let $I$ be a proper ideal of $R$, and let $n \geq 1$ be a positive integer. In this paper, we introduce a class of ideals that is closely related to the class of $I$-prime ideals. A proper ideal $P$ of $R$ is called an $n$-absorbing $I$-ideal if $a_1, a_2, \ldots, a_{n+1} \in R$ with $a_1a_2 \ldots a_{n+1} \in P - IP$, then $a_1a_2 \ldots a_{i-1}a_{i+1} \ldots a_{n+1} \in P$ for some $i \in \{1, 2, \ldots, n+1\}$. Among many results, we show that every proper ideal of a ring $R$ is an $n$-absorbing $I$-ideal if and only if every quotient of $R$ is a product of $(n+1)$-fields.

1. Introduction

Throughout this article, $R$ denotes a commutative ring with identity and $\text{Max}(R)$ denotes the set of all maximal ideals of $R$. The notion of prime ideal plays a main role in the theory of commutative algebra and it has been widely studied and recently many generalizations were introduced by many authors. Recall from [4] that a prime ideal of $R$ is a proper ideal $P$ with the property that for $a, b \in R$, $ab \in P$ implies $a \in P$ or $b \in P$. The concept of $I$-prime ideals was defined and investigated in [1]. For a fixed ideal $I$ of $R$, a proper ideal $P$ of $R$ is $I$-prime if $a, b \in R$ with $ab \in P - IP$ implies either $a \in P$ or $b \in P$. The concept of 2-absorbing ideals was introduced and studied in [5]. Let $n$ be a positive integer. A proper ideal $P$ of a ring $R$ is called an $n$-absorbing ideal if whenever $x_1 \ldots x_{n+1} \in P$ for $x_1, \ldots, x_{n+1} \in R$, then there are $n$ of the $x_i$’s whose product is in $P$. Equivalently, a proper ideal $P$ of $R$ is an $n$-absorbing ideal if and only if whenever $x_1 \ldots x_m \in P$ for $x_1, \ldots, x_m \in R$ with $m > n$, then there are $n$ of the $x_i$’s whose product is in $P$; see [3].

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Let \( n \geq 2 \) and let \( \phi : S(R) \to S(R) \cup \{ \phi \} \) be a map, where \( S(R) \) is the set of ideals of \( R \). A proper ideal \( P \) of \( R \) is called \((n - 1, n)\)-\( \phi \)-prime if whenever \( a_1, a_2, \ldots, a_n \in R \) with \( a_1a_2 \ldots a_n + P = \phi(P) \), then the product of \( n - 1 \) of the \( a_i \)'s is in \( P \) (see [6]).

In this article, we introduce a class of ideals that is closely related to the class of I-Prime ideals. Let \( I \) be a proper ideal of \( R \) and let \( n \geq 1 \). A proper ideal \( P \) of \( R \) is called an \( n \)-absorbing I-prime ideal of \( R \) if \( a_1, a_2, \ldots, a_{n+1} \in R \) with \( a_1a_2 \ldots a_{n+1} + P = IP \), then \( a_1a_2 \ldots a_{i-1}a_{i+1} \ldots a_{n+1} \in P \) for some \( i \in \{1, 2, \ldots, n + 1\} \). Thus a \( 1 \)-absorbing I-ideal is just an I-prime ideal. If we set \( \phi(P) = IP \) for every \( P \) in \( S(R) \), then the ideas of this paper are a special case of the paper [6]. Some properties of the \( n \)-absorbing I-prime ideals are discussed and studied.

2. Main results

Let \( I \) be a fixed ideal of a ring \( R \) and let \( n \geq 1 \) be a positive integer. A proper ideal \( P \) of \( R \) is called an \( n \)-absorbing I-ideal if \( a_1, a_2, \ldots, a_{n+1} \in R \) with \( a_1 \ldots a_{n+1} + 1 \in P - IP \), then \( a_1 \ldots a_{i-1}a_{i+1} \ldots a_{n+1} + 1 \in P \) for some \( i \in \{1, 2, \ldots, n + 1\} \). It is clear that a proper ideal \( P \) is an \( n \)-absorbing I-ideal of \( R \) if and only if whenever the product of \( n+1 \)-element of \( R/P \) is 0, then the product of some \( n \) of these elements is 0 in \( R/P \). Not all \( n \)-absorbing I-ideals are \((n-1)\)-absorbing I-ideal. The following example illustrates this fact.

**Example 2.1.** Consider the ring \( R = k[[x, y]]/ \langle x^n, y^n, x^{2n} - y^{2n}, x^{2n+1}y^{2n+1} \rangle \), where \( k \) is a field and \( n \geq 1 \) is a positive integer. Put the fixed ideal \( I \) to be zero ideal of \( R \). Then the proper ideal \( P = \langle \bar{x}^n, \bar{y}^n, \bar{x}^{2n} - \bar{y}^{2n}, \bar{x}^{2n+1}\bar{y}^{2n+1} \rangle \) of \( R \) is a \((2n+1)\)-absorbing I-ideal but not a \( 2n \)-absorbing I-ideal, since \( \bar{x}^{2n} \in P \) and \( \bar{x}^{2n+1} \notin P \).

The proof of the following lemma comes directly from the definition so it is omitted.

**Lemma 2.2.** A proper ideal \( P \) of a ring \( R \) is an \( n \)-absorbing I-ideal if and only if \( \frac{P}{IP} \) is an \( n \)-absorbing 0-ideal.

**Proposition 2.3.** Let \( P \) be an \((n+1)\)-ideal of a ring \( R \) and let \( S \subseteq R \) be a multiplicative closed set of \( R \) such that \( P \cap S = \phi \). Then \( S^{-1}P \) is an \( n \)-absorbing \( S^{-1}I \)-ideal of \( S^{-1}R \).

**Proof.** Suppose \( \frac{a_1}{s_1}, \ldots, \frac{a_{n+1}}{s_{n+1}} \in S^{-1}R \) with \( \frac{a_1a_2 \ldots a_{n+1}}{s_1s_2 \ldots s_{n+1}} \in S^{-1}P - S^{-1}IS^{-1}P = S^{-1}(P - IP) \). Then \( ua_1a_2 \ldots a_{n+1} \in P \) for some \( u \in S \). By taking \( ua_1 \) as one element, either \( a_2 \ldots a_{n+1} \in P \) or \( ua_1 \ldots a_{i-1}a_{i+1} \ldots a_{n+1} \in P \) for \( i = 2, 3, \ldots, n + 1 \). Hence \( \frac{a_2 \ldots a_{i+1}}{s_2 \ldots s_{n+1}} \cdot \frac{a_{i+1} \ldots a_{n+1}}{s_{i+1} \ldots s_{n+1}} \in S^{-1}P \) or \( \frac{ua_1 \ldots a_i-1a_{i+1} \ldots a_{n+1}}{s_1 \ldots s_i-1s_{i+1} \ldots s_{n+1}} \in S^{-1}P \), which means that \( S^{-1}P \) is an \( n \)-absorbing \( S^{-1}I \)-ideal of \( S^{-1}R \). \( \square \)

**Theorem 2.4.** Let \( P \) be a proper ideal of a commutative ring \( R \). If \( P \) is an \( n \)-absorbing I-ideal that is not an \( n \)-absorbing ideal, then \( P^{n+1} \subseteq IP \).
Proof. Assume that $P^n \notin IP$. We have to show that $P$ is an $n$-absorbing ideal. Let $x_1x_2 \ldots x_{n+1} \in P$ for $x_1$, $x_2$, \ldots, $x_{n+1} \in R$. If $x_1x_2 \ldots x_{n+1} \notin IP$, then the $n$-absorbing $I$-ideal $P$ gives that $P$ is an $n$-absorbing ideal. Now, for the case $x_1x_2 \ldots x_{n+1} \in IP$, we have $x_1x_2 \ldots x_{n+1-k}P^k \subseteq IP$ for $k = 1$, 2, \ldots, $n$, since otherwise, we obtain $x_1P_2 \ldots x_{n+1-k}P_k \notin IP$ for $p_1$, $p_2$, \ldots, $p_k \in P$ and so $x_1x_2 \ldots x_{n+1-k} (x_{n+2-k} + p_1) \ldots (x_{n+1} + p_k) \in P - IP$. As $P$ is an $n$-absorbing $I$-ideal, $x_1x_2 \ldots x_{i-1}x_{i+1} \ldots x_{n+1} \in P$, for some $i = \{1, 2, \ldots, n+1\}$. Similarly, we can assume that for all $i_1, i_2, \ldots, i_{n+1-k} \subseteq \{1, 2, \ldots, n+1\}$, $a_{i_1} \ldots a_{i_{n+1-k}} P^k \subseteq IP$ with $1 \leq k \leq n+1$. Since $P^{n+1} \notin IP$, there exist $r_1, r_2, \ldots, r_{n+1} \in P$ with $r_1r_2 \ldots r_{n+1} \notin IP$. Then $(x_1 + r_1)(x_2 + r_2) \ldots (x_{n+1} + r_{n+1}) \in P - IP$. Thus being $P$ $n$-absorbing $I$-ideal gives us $x_1x_2 \ldots x_{i-1}x_{i+1} \ldots x_{n+1} \in P$ for some $i \in \{1, 2, \ldots, n+1\}$. Therefore $P$ is an $n$-absorbing ideal.

We conclude from Theorem 2.4 that an $n$-absorbing $I$-ideal $P$ with $P^{n+1} \notin IP$ is an $n$-absorbing ideal.

**Corollary 2.5.** Let $R$ be a ring and let $P$ be a proper ideal of $R$. If $P$ is an $n$-absorbing 0-ideal that is not an $n$-absorbing ideal, then $P^{n+1} = 0$.

**Corollary 2.6.** Let $P$ be an $n$-absorbing $I$-ideal with $(IP) \subseteq P^{n+2}$. Then $P$ is an $n$-absorbing $\cap_{i=1}^{\infty} P^i$-ideal ($n \geq 1$).

**Proof.** If $P$ is an $n$-absorbing ideal, then $P$ is an $n$-absorbing $I$-ideal and so is an $n$-absorbing $\cap_{i=1}^{\infty} P^i$-ideal. Suppose that $P$ is not a an $n$-absorbing ideal, then Theorem 2.4 gives us $P^{n+1} \subseteq IP \subseteq P^{n+2}$. Hence $IP = P^k$ for each $k \geq n + 1$ and hence $\cap_{i=1}^{\infty} P^i = IP$. Thus $P$ is an $n$-absorbing $\cap_{i=1}^{\infty} P^i$-ideal. □

Let $R$ and $S$ be two rings. If $P$ is an $n$-absorbing 0-ideal of $R$. Then $P \times S$ need not be an $n$-absorbing 0-ideal of $R \times S$. For a particularly case see [2, Theorem 7]. However, $P \times S$ is an $n$-absorbing $I$-ideal for each $I$ with $\cap_{i=1}^{\infty} (P \times S)^i \subseteq I(P \times S) \subseteq P \times S$.

**Theorem 2.7.** (1) Let $R$ and $S$ be two rings and let $P$ be an $n$-absorbing 0-ideal of $R$. Then $J = P \times S$ is an $n$-absorbing $I$-ideal of $R \times S$, for each $I$ with $\cap_{i=1}^{\infty} (P \times S)^i \subseteq I(P \times S) \subseteq P \times S$.

(2) Let $R$ be a commutative ring and let $J$ be a finitely generated proper ideal of $R$. Suppose that $J$ is an $n$-absorbing $I$-ideal, where $IP \subseteq J^{n+2}$. Then either $J$ is an $n$-absorbing 0-ideal or $J^{n+1} \neq 0$ is idempotent and $R$ decomposes as $T \times S$, where $S = J^{n+1}$ and $J = P \times S$, where $P$ is an $n$-absorbing 0-ideal. Hence $J$ is an $n$-absorbing $I$-ideal for each $I$ with $\cap_{i=1}^{\infty} J^i \subseteq IJ \subseteq J$.

**Proof.** (1) Let $R$ and $S$ be two rings and let $P$ be an $n$-absorbing 0-ideal of $R$. Then $P \times S$ need not be an $n$-absorbing 0-ideal of $R \times S$. In fact, $P \times S$ is an $n$-absorbing 0-ideal if and only if $P \times S$ is a prime ideal. However, $P \times S$ is an $n$-absorbing $I$-ideal for each $I$ with $\cap_{i=1}^{\infty} (P \times S)^i \subseteq I(P \times S)$. If $P$ is an $n$-absorbing ideal, then $P \times S$ is an $n$-absorbing ideal and thus is an $n$-absorbing $I$-ideal. Assume that $P$ is not an $n$-absorbing ideal. Then $P^{n+1} = 0$ and $(P \times S)^{n+1} = 0 \times S$. Hence $\cap_{i=1}^{\infty} (P \times S)^i = \cap_{i=1}^{\infty} P^i \times S = 0 \times S$. Thus $P \times S \cap_{i=1}^{\infty} (P \times S)^i =$
$P \times S - 0 \times S = (P - 0) \times S$. Since $P$ is an $n$-absorbing 0-ideal, $P \times S$ is an $n$-absorbing $\cap_{i=1}^{n}(P \times S)^i$-ideal and as $\cap_{i=1}^{n}(P \times S)^i \subseteq I(P \times S)$, $P \times S$ is an $n$-absorbing $I$-ideal.

(2) If $J$ is an $n$-absorbing ideal, then $J$ is an $n$-absorbing 0-ideal. So, we can assume that $J$ is not an $n$-absorbing ideal. Then $J^{n+1} \subseteq IP$ and hence $J^{n+1} \subseteq IP \subseteq J^{n+2}$, so $J^{n+1} = J^{n+2}$. Hence $J^{n+1}$ is idempotent. Since $J^{n+1}$ is finitely generated, $J^{n+1} = (e)$ for some idempotent $e \in R$. Suppose $J^{n+1} = 0$. Then $IP = 0$, and hence $J$ is an $n$-absorbing 0-ideal. Assume that $J^{n+1} \neq 0$, and put $S = J^{n+1} = Re$ and $T = R(1 - e)$, so $R$ decomposes $T \times S$. Let $P = J(1 - e)$; so $J = P \times S$, where $P^{n+1} = (J(1 - e))^{n+1} = J^{n+1}(1 - e)^{n+1} = (e)(1 - e) = 0$. We claim that $P$ is an $n$-absorbing 0-ideal. Let $x_1$, $x_2$, ..., $x_n+1 \in R$ and let $0 \neq x_1x_2...x_{n+1} \in P$. Then $(x_1, 0)(x_2, 0)...(x_{n+1}, 0) = (x_1x_2...x_{n+1}, 0) \in P \times S - (P \times S)^{n+1} = P \times S - 0 \times S \subseteq P - IP$, since $IP \subseteq J^{n+2}$, which implies that $IP \subseteq J^{n+2} = (P \times S)^{n+2} = 0 \times S$. Hence $J - J^{n+1} \subseteq J - IP$. As $J$ is an $n$-absorbing $I$-ideal, $(x_1x_2...x_{i-1}x_{i+1}...x_{n+1}, 0) \in P \times S = J$, for some $i \in \{1, 2, ..., n + 1\}$. Thus $x_1x_2...x_{i-1}x_{i+1}...x_{n+1} \in P$. Hence $P$ is an $n$-absorbing 0-ideal.}

**Corollary 2.8.** Let $R$ be an indecomposable ring and let $P$ be a finitely generated $n$-absorbing $I$-ideal of $R$, where $IP \subseteq P^{n+2}$. Then $P$ is an $n$-absorbing 0-ideal. Furthermore, if $R$ is an integral domain, then $P$ is actually an $n$-absorbing ideal.

**Corollary 2.9.** A proper ideal $P$ of a Noetherian integral domain $R$ is an $n$-absorbing ideal if and only if $P$ is an $n$-absorbing $P^{n+1}$-ideal for $(n \geq 2)$.

In what follows, we characterize an $n$-absorbing $I$-ideals.

**Theorem 2.10.** Let $P$ be a proper ideal of a ring $R$. Then the following conditions are equivalent.

(1) $P$ is an $n$-absorbing $I$-ideal.

(2) For $x_1, x_2, ..., x_n \in R - P$:

$$P : x_1x_2...x_n = \cup_{i=1}^{n} (P : x_1x_2...x_{i-1}x_{i+1}...x_n) \cup (IP : x_1x_2...x_n).$$

**Proof.** (1) $\Rightarrow$ (2) Suppose $x_1, x_2, ..., x_n \in R - P$ and $y \in (P : x_1x_2...x_n)$. Then $x_1x_2...x_ny \in P$. If $x_1x_2...x_ny \notin IP$, then $x_1x_2...x_{i-1}x_{i+1}...x_ny \in P$, for some $i \in \{1, 2, ..., n\}$, and so $y \in (P : x_1x_2...x_{i-1}x_{i+1}...x_n)$. If $x_1x_2...x_ny \in IP$, then $y \in (IP : x_1x_2...x_n)$. Hence

$$P : x_1x_2...x_n \subseteq \cup_{i=1}^{n} (P : x_1x_2...x_{i-1}x_{i+1}...x_n) \cup (IP : x_1x_2...x_n).$$

The other containment always holds.

(2) $\Rightarrow$ (1) Suppose $x_1x_2...x_{n+1} \in P - IP$. If $x_1x_2...x_n \in P$, then there is nothing to prove. Assume that $x_1x_2...x_n \notin P$. Thus

$$P : x_1x_2...x_n = \cup_{i=1}^{n} (P : x_1x_2...x_{i-1}x_{i+1}...x_n) \cup (IP : x_1x_2...x_n).$$

Since $x_1x_2...x_{n+1} \in P$, $x_{n+1} \in (P : x_1x_2...x_n)$ and the fact $x_1x_2...x_{n+1} \notin IP$ gives us $x_{n+1} \notin (IP : x_1x_2...x_n)$. Hence $x_{n+1} \in (P : x_1x_2...x_{i-1}x_{i+1}...x_n)$, for some $i \in \{1, 2, ..., n\}$, that is, $x_1x_2...x_{i-1}x_{i+1}...x_{n+1} \in P$. Thus $P$ is an $n$-absorbing $I$-ideal. □
It was shown by Anderson and Smith [2, Theorem 8] that every proper ideal of $R$ is weakly prime if and only if $R$ is a direct product of two fields or $(R,m)$ is quasi-local with $M^2 = 0$. Next we generalize this result to an $n$-absorbing $I$-ideals but first we need the following lemma.

**Lemma 2.11.** Let $R = R_1 \times R_2 \times \cdots \times R_{n+1}$, where $R_i$ is a ring, for $i \in \{1, 2, \ldots, n+1\}$. If $P$ is an $n$-absorbing $I$-ideal of $R$, then either $P = IP$ or $P = P_1 \times P_2 \times \cdots \times P_{i-1} \times R_i \times P_{i+1} \times \cdots \times P_{n+1}$ for some $i \in \{1, 2, \ldots, n+1\}$ and if $P_j \neq R_i$ for $j \neq i$, then $P_j$ is an $n$-absorbing ideal in $R_j$.

**Proof.** Let $P = P_1 \times P_2 \times \cdots \times P_{n+1}$ be an $n$-absorbing $I$-ideal of $R$. Then there exists $(x_1, x_2, \ldots, x_{n+1}) \in P - IP$, and so

$$(x_1, 1, \ldots, 1)(1, x_2, 1, \ldots, 1) \cdots (1, 1, \ldots, x_{n+1}) = (x_1, x_2, \ldots, x_{n+1}) \in P.$$ 

As $P$ is an $n$-absorbing $I$-ideal, we have $(x_1, x_2, \ldots, x_{i-1}, 1, x_{i+1}, \ldots, x_{n+1}) \in P$ for some $i \in \{1, 2, \ldots, n+1\}$. Thus $(0, 0, \ldots, 0, 1, 0, \ldots, 0) = P = P_1 \times P_2 \times \cdots \times P_{i-1} \times R_i \times P_{i+1} \times \cdots \times P_{n+1}$. If $P_j \neq R_i$ for $j \neq i$, then we have to prove $P_j$ is an $n$-absorbing ideal in $R_j$. Let $i < j$ and let $y_1y_2 \ldots y_{n+1} \in P_j$. Then

$$(0, 0, \ldots, 0, 1, 0, \ldots, 0, y_1y_2 \ldots y_n, 0, \ldots, 0) = (0, 0, \ldots, 1, 0, \ldots, 0)(0, 0, \ldots, 1, 0, \ldots, y_2, 0) \ldots (0, 0, \ldots, 1, 0, \ldots, y_{n+1}, 0, \ldots, 0) \in P - IP$$

and the $n$-absorbing $I$-ideal $P$ give that

$$(0, 0, \ldots, 0, 1, 0, \ldots, 0, y_1y_2 \ldots y_{k-1}y_{k+1} \ldots y_{n+1}, 0, \ldots, 0) \in P$$

for some $k \in \{1, 2, \ldots, n+1\}$. Thus $y_1y_2 \ldots y_{k-1}y_{k+1} \ldots y_{n+1} \in P_j$ and hence $P_j$ is an $n$-absorbing ideal in $R_j$. We can do the same arguments for the case $j < i$. □

**Theorem 2.12.** Let $R$ be a ring and let $|\text{Max}(R)| \geq n + 1 \geq 2$. Every proper ideal of $R$ is an $n$-absorbing $I$-ideal if and only if every quotient of $R$ is a product of $(n+1)$-fields.

**Proof.** ($\Leftarrow$): Let $P$ be a proper ideal of $R$. Then $R \cong F_1 \times F_2 \times \cdots \times F_{n+1}$ and $P \cong P_1 \times P_2 \times \cdots \times P_{n+1}$, where $P_i$ is an ideal of $F_i$, $i = 1, 2, \ldots, n + 1$. If $P = IP$, then there is nothing to prove, otherwise we have $P_j = 0$ for at least one $j \in \{1, 2, \ldots, n+1\}$, since $P \cong F_i$ is proper. Therefore $P \cong F_i$ is an $n$-absorbing 0-ideal of $R$ and $P$ is an $n$-absorbing $I$-ideal of $R$.

($\Rightarrow$): Let $m_1, m_2, \ldots, m_{n+1}$ be distinct maximal ideals of $R$. Then $m = m_1m_2 \ldots m_{n+1}$ is an $n$-absorbing $I$-ideal of $R$. We want to show that $m$ is not an $n$-absorbing ideal. First to show that $m_i \not\subseteq \cup_{j \neq i} m_j$ for all $i \in \{1, 2, \ldots, n + 1\}$, we suppose the contrary that $m_i \subseteq \cup_{j \neq i} m_j$. Then there exists $m_j$ with $m_i \subseteq m_j$ by prime avoidance lemma, which contradicts the fact that $m_i$, $i = 1, 2, \ldots, n + 1$ are distinct maximal ideals. Hence there exists $x_i \in m_i - \cup_{i \neq j=1}^{n+1} m_j$ and so $x_1, x_2, \ldots, x_{n+1} \subseteq m$. If there exists $j \in \{1, 2, \ldots, n + 1\}$ with $x_1x_2 \ldots x_{j-1}x_{j+1} \ldots x_{n+1} \in m \subseteq m_j$, then $x_i \in m_j$, for some $i \neq j$, a contradiction. Hence $m$ is not an $n$-absorbing ideal and so $m^{n+1} = Im$. Thus by the
Chinese remainder theorem, $\frac{R}{Im} \cong \frac{R}{m_1^{n+1}} \times \frac{R}{m_2^{n+1}} \times \cdots \times \frac{R}{m_{n+1}^{n+1}}$. Put $F_i = \frac{R}{m_i^{n+1}}$. If $F_i$ is not field, then it has a nonzero proper ideal $K$ and so $0 \times 0 \times \cdots \times 0 \times K \times 0 \times \cdots \times 0$ is an $n$-absorbing 0-ideal of $\frac{R}{Im}$. Thus by Lemma 2.11, we have $K = F_i$ or $K = 0$, which is impossible. Hence $F_i$ is a field. □

Corollary 2.13. Let $R$ be a ring and let $|\text{Max}(R)| \geq n + 1 \geq 2$. Every proper ideal of $R$ is an $n$-absorbing 0-ideal if and only if $R \cong F_1 \times F_2 \times \cdots \times F_{n+1}$, where $F_1, F_2, \ldots, F_{n+1}$ are fields.

In what follows, we characterize rings with the property that every proper ideal is an $n$-absorbing 0-ideal.

Corollary 2.14. Let $P$ be an $n$-absorbing $I$-ideal of a ring $R$, where $IP \subseteq P^{n+2}$. Then $P$ is an $n$-absorbing $\cap_{i=1}^{\infty} P^i$-ideal $(n \geq 2)$.

Proof. If $P$ be an $n$-absorbing ideal, then $P$ is an $n$-absorbing $I$-ideal and so is an $n$-absorbing $\cap_{i=1}^{\infty} P^i$-ideal. Suppose that $P$ is not an $n$-absorbing ideal. Then Theorem 2.4 gives us $P^{n+1} \subseteq IP \subseteq P^{n+2}$. Hence $IP = P^k$ for each $k \geq n + 1$ and hence $\cap_{i=1}^{\infty} P^i = IP$. Thus $P$ is an $n$-absorbing $\cap_{i=1}^{\infty} P^i$-ideal. □

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