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SELF-CENTERED GRAPHS WITH DIAMETER 3

ELHAM IMANI[1] AND MADJID MIRZAVAZIRI[1*]

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Abstract. A graph is called 3-self-centered if all of its vertices have eccentricity three. In this article, we study some properties of the 3-self-centered graphs and investigate the 3-self-centered graphs with girth six or seven. We determine the vertex connectivity and regularity of such graphs. Moreover, we show that if G is a 3-self-centered graph with girth seven, then G is regular.

1. Introduction and preliminaries

In this article, we only consider finite undirected connected simple graphs. For a vertex v of a graph G, the *eccentricity* of v, denoted by $\mathrm{e}(v)$, is the maximum distance between v and other vertices of G. The maximum and minimum of eccentricities among all vertices of G are called the *diameter* and the *radius* of G, denoted by $\mathrm{diam}(G)$ and $\mathrm{rad}(G)$, respectively. The *center* of G is the set of all vertices with the minimum eccentricity. A graph G is called self-centered if its center consists of all its vertices and is called d-self-centered if $\mathrm{diam}(G) = \mathrm{rad}(G) = d$. For brevity, we show d-self-centered by d-sc. Obviously, G is a 1-sc graph if and only if G is a complete graph. In [8], it was characterized the edge-minimal 2-sc graphs. Stanic [9] investigated minimal self-centered graphs. For the relation between self-centered graphs and chordal graphs, see [1, 4]. For more information, we refer the reader to the articles [3, 5–7].

We denote the set of all vertices and edges of G by $V(G)$ and $E(G)$, respectively. For a vertex $v \in V(G)$, the number of incident edges to it is called *degree* of v, and is denoted by $\mathrm{d}(v)$. The minimum and maximum degrees of the vertices of G are shown by $\delta(G)$ and $\Delta(G)$, respectively. The graph G is called *r-regular* if the degree of every vertex of G is r.

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*Corresponding author.
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If two vertices $u, v \in V(G)$ are adjacent, then we write $u \sim v$. For $u, v \in V(G)$, $\mathrm{d}(u, v)$ denotes the length of the shortest path between u and v. If there is no path between u and v, then $\mathrm{d}(u, v) = \infty$. For an integer i, we define $N_i(v) = \{u \in V(G) | \mathrm{d}(u, v) = i\}$. For a path P, the *length* of P is defined as the number of its edges and is denoted by $\ell(P)$. The *girth* of G is the length of shortest cycle of G and is denoted by *girth*(G). If G has no cycle, then the girth of G is infinite. An (r, g)-*cage* graph is an r-regular graph of girth g with minimum possible number of vertices. A *Moore* graph is a regular graph of diameter d and girth $2d + 1$.

We remind that a uv-path is a path between u and v. Two uv-paths P and Q are *internally disjoint* if $V(P) \cap V(Q) = \{u, v\}$. If G is a connected graph with at least k vertices and by removing any arbitrary $k - 1$ vertices of G, it remains connected, then G is called k-*connected*. If G is k-connected and is not $(k + 1)$-connected, then the *connectivity* of G is defined to be k. We know that if for any two distinct vertices u and v, there are at least k internally disjoint paths between u and v, then G is k-connected.

There are many publications on 2-sc graphs; see, for example, [3, 5, 7, 8]. Unfortunately, 3-sc graphs are not investigated yet. In this article, we focus on 3-sc graphs and find some properties of these graphs. In section 2, we study some basic properties of 3-sc graphs. In particular, we find the girth of graph and show that a 3-sc graph has a cycle of length six or seven. In Section 3, we investigate the connectivity of 3-sc graphs with girth six or seven, and show that the vertex connectivity is at least $\delta(G)$. Moreover, we show that if G is a 3-sc graph with girth seven, then G is regular.

2. Some instant observations

In this section, we discover some basic properties of 3-sc graphs. Let us start with the most important one, that is, the vertex degrees.

Proposition 2.1. *Let G be a 3-sc graph on n vertices and let $v \in V(G)$. Then $n \geq 6$, G is at least 2-connected and we have $2 \leq \mathrm{d}(v) \leq n - 4$.*

Proof. First we show that $n \geq 6$. Since G is a 3-sc graph, there are two vertices $u, v \in V(G)$ such that $\mathrm{d}(u, v) = 3$. Assume that $u \sim w \sim z \sim v$ is a path with length three between u and v. Since G is a 3-sc graph, there must be at least two other vertices with distance three from w and z.
Now, we prove that G is a 2-connected graph. On contrary, assume that G has a cut vertex, say u. Therefore, by removing u from G, the new graph has at least two components. Since G is a 3-sc graph, there must be a vertex with distance three from u, say v. Suppose that $u \sim w \sim z \sim v$ is a path between u and v. In this case, the vertices w, z, and v belong to one component. Now, select one vertex from other component, say x. Hence, $\mathrm{d}(v, x) > 3$, which is a contradiction. Since G is a 3-sc graph and G is a 2-connected, we deduce that $2 \leq \mathrm{d}(v)$, for $v \in V(G)$. Assume that there exists a vertex $z \in V(G)$ such that $\mathrm{d}(z) > n - 3$. If $\mathrm{d}(z) = n - 1$, then $\mathrm{rad}(G) = 1$. Now, let $\mathrm{d}(z) = n - 2$ and let $v \notin N(z)$. Then $\mathrm{rad}(G) \leq 2$, which is a contradiction. If $\mathrm{d}(z) = n - 3$, then $|V(G) \setminus N(z)| = 2$. Suppose that $u, v \in V(G) \setminus N(z)$. Since $\mathrm{d}(z, v) = \mathrm{d}(z, u) = 2$ leads a contradiction, we assume that $\mathrm{d}(z, v) = 3$ and that $\mathrm{d}(z, u) = 2$. Therefore, there is $y \in V(G)$

such that $z \sim y \sim u \sim v$. On the other hand, for each vertex $w \in V(G)$, $\mathrm{d}(w,y) < 3$, which is a contradiction. □

Another, would be the girth, as we see below.

Proposition 2.2. *Let G be a 3-sc graph. Then $3 \leq \mathrm{girth}(G) \leq 7$.*

Proof. Since G is connected and $\mathrm{d}(v) \geq 2$ for each vertex v, then G has to contain a cycle. The lower bound is clear (and attainable by the 3-sc graph H_1 in Fig. 1), so we are only required to prove the upper bound. On contrary if there is a 3-sc graph G with $\mathrm{girth}(G) \geq 8$, then, for each vertex of a cycle of length equal to $\mathrm{girth}(G)$, there is another vertex whose distance from the first is more than three. □

It must be reminded that the above proposition presents tight bounds on the girth of a 3-sc graph; while C_7 is itself a 3-sc graph with girth seven, the graph H_1 (and also the graph H_2) drawn bellow is a 3-sc graph with girth three.

Proposition 2.3. *The only 3-sc graph on six vertices is C_6.*

Proof. It is easy to check that C_6 is a 3-sc graph. To show the uniqueness, let G be a 3-sc graph on six vertices, and let $\{v_1, \dots v_6\}$ be its set of vertices. Then for v_1, there is at least a vertex whose distance from it is three. Without loss of generality, suppose that this vertex is v_4, and we have the path $v_1 \sim v_2 \sim v_3 \sim v_4$ as an induced subgraph. By Proposition 2.1, we have $\mathrm{d}(v_4) \geq 2$, and since it cannot be adjacent to v_1 or v_2, there must be a vertex in $V(G) \setminus \{v_1, v_2, v_3, v_4\}$, namely v_5, such that it is adjacent to v_4. The vertex v_5 cannot be adjacent to v_1 because if it is, then the distance between v_1 and v_4 becomes less than 3. While $\mathrm{d}(v_1) \geq 2$, there must be a vertex other than $v_2, \dots, v_5$ that v_1 is adjacent to. Therefore, it has to be v_6.

Note that by Proposition 2.1, $\mathrm{d}(v_5)$ and $\mathrm{d}(v_6)$ are both greater than two. If v_5 is adjacent to v_2 or v_3, then there is no vertex in $V(G) \setminus \{v_2\}$ whose distance from v_2 is three. With the same reasoning, we can deduce that v_6 is adjacent to neither v_2 nor v_3. Therefore, the only possibility is that v_5 and v_6 are adjacent.

Consequently, we have C_6 as a subgraph of G. Since any other adjacency reduces the radius of G, it can be deduced that $G \simeq C_6$. □

Proposition 2.4. *Every 3-sc graph contains a cycle of length six or seven.*

Proof. Suppose that G is a 3-sc graph and that $u \in V(G)$. There exists a vertex $v \in V(G)$ such that $\mathrm{d}(u,v) = 3$. Assume that P a path $u \sim w \sim z \sim v$. Now, select $t \in N(u)$ such that $t \neq w$. Since G is a 3-sc graph, there is a path Q between t and v with the length at least two and at most three. Hence, the paths P, Q, and $u \sim t$ form a cycle of length six or seven. □

Now, we characterize the 3-sc graphs with seven vertices.

Proposition 2.5. *The only 3-sc graphs on seven vertices are C_7, H_1, H_2, and H_3 (Fig. 1).*

Proof. Suppose that G is a 3-sc graph on seven vertices. By Proposition 2.4, G has a cycle of length six or seven. Now, consider the following two cases:

Case 1. G has a cycle of length six. Assume that vertices $v_1, \ldots, v_6$ are on a cycle C of length six. Clearly, G has another vertex, namely v_7. Since G is a 2-connected graph, the degree of v_7 is at least two. If $\mathrm{d}(v_7) \geq 4$, then $\mathrm{e}(v_7) \leq 2$. Hence, we can assume that the degree of v_7 is either two or three. First, suppose that $\mathrm{d}(v_7) = 2$. In this case, if v_7 is adjacent to two adjacent vertices of C, then G is isomorphic to H_1. If v_7 is adjacent to two vertices of C with distance two, then G is isomorphic to H_3. Second, suppose that $\mathrm{d}(v_7) = 3$. Therefore, v_7 is adjacent to three consecutive vertices of C and G is isomorphic to H_2.
Case 2. G has no cycle of length six. Therefore G has a cycle of length seven. Since G has seven vertices, we can arrange the vertices in a Hamiltonian cycle. If G has a vertex of degree three, then there exists a cycle of length less than seven, which is a contradiction. Thus G is isomorphic to C_7. □

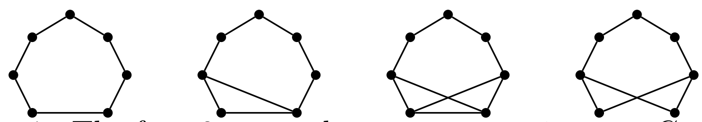

FIGURE 1. The four 3-sc graphs on seven vertices are C_7, H_1, H_2, and H_3, respectively.

3. Connectivity and regularity

In the next theorem, we show that the 3-sc graphs of girth seven are δ-connected.

Theorem 3.1. *Every 3-sc graph G with girth seven is $\delta(G)$-connected.*

Proof. We show that for every two vertices $u, v \in V(G)$, there are $k = \delta(G)$ internally disjoint uv-paths. Since $\mathrm{d}(u, v) \leq 3$, we consider the following three cases:
Case 1. Suppose that $\mathrm{d}(u, v) = 1$. Let $N_1(u) = \{v, u_2, \ldots, u_k\}$ and $N_1(v) = \{u, v_2, \ldots, v_s\}$, with $k \leq s$. Proposition 2.1 implies that G has no leaf. Hence, because $girth(G) = 7$, there exist $k - 1$ vertices $u'_i \in N_1(u_i) \setminus \{u\}$, for $2 \leq i \leq k$; see Fig. 2.

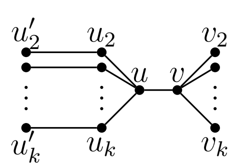

FIGURE 2. The path between v_i and u'_i, for $2 \leq i \leq k$.

Let Q_i be the path $u'_i \sim u_i \sim u \sim v \sim v_i$. Since $\mathrm{girth}(G) = 7$, we have $\mathrm{d}(u'_i, v_i) = 3$. Let P_i be the path of length three between u'_i and v_i, for each i, $2 \leq i \leq k$. If there exists integer i ($2 \leq i \leq k$) such that $V(P_i) \cap V(Q_i) \neq \{u'_i, v_i\}$, then there is a cycle of length smaller than seven. Thus, $V(P_i) \cap V(Q_i) = \{u'_i, v_i\}$, for each i, $2 \leq i \leq k$. It is not difficult to see that $V(P_i) \cap V(P_j) = \emptyset$, for

$i \neq j$. Hence, $P_i \cup Q_i$ forms a cycle of length seven. Now, by removing the edge $uv \in E(G)$, we have $k-1$ internally disjoint paths between u and v. On the other hand, the $u \sim v$ is a path of length one. So, there are k internally disjoint uv-paths.

Case 2. Assume that $\mathrm{d}(u,v) = 2$. Let $N_1(u) = \{w, u_2, \ldots, u_k\}$ and let $N_1(v) = \{w, v_2, \ldots, v_s\}$, see Fig. 3.

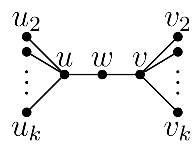

FIGURE 3. The path between v_i and u_i, for $2 \leq i \leq k$.

Since $\mathrm{d}(u_i, v_i) \leq 3$ and $\mathrm{girth}(G) = 7$, we have $\mathrm{d}(u_i, v_i) = 3$. Let P_i be the path of length three between u_i and $v_i, 2 \leq i \leq k$. Clearly, $V(P_i) \cap V(P_j) = \emptyset$, for $i \neq j$. Each $P_i \cup Q_i$ forms a cycle of length seven. Again, by removing the path $u \sim w \sim v$, we have $k-1$ paths between u and v. On the other hand, the $u \sim w \sim v$ is a path of length two. So, there are at least k internally disjoint uv-paths.

Case 3. Consider $\mathrm{d}(u,v) = 3$. Let $u \sim w \sim w' \sim v$ and let $N_1(u) = \{w, u_2, \ldots, u_k\}$, see Fig. 4

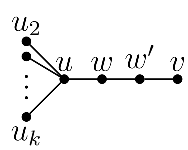

FIGURE 4. The path between v and u_i, for $2 \leq i \leq k$.

Then $\mathrm{d}(u_i, v) = 3$, for every i, $2 \leq i \leq k$. Hence, assume that P_i is a path of length three between u_i and v, $2 \leq i \leq k$. Clearly, $V(P_i) \cap V(P_j) = \emptyset$, for $i \neq j$. In this case, $P_i \cup \{u \sim u_i\}$ are $k-1$ internally disjoint paths between u and v. □

In the next theorem, we characterize the specific 3-sc graphs with girth seven.

Theorem 3.2. *Let G be a 3-sc graph with girth seven and $\delta(G) = 2$. Then $G \simeq C_7$.*

Proof. If all vertices of G have degree two, then $G \simeq C_7$. Thus, assume that G has a vertex of degree at least three. Now, consider the following two cases:

Case 1. There are two adjacent vertices of degree two. Suppose that $u_1 \sim u_2 \sim u_3 \sim u_4$, $\mathrm{d}(u_1) \geq 3$, and that $\mathrm{d}(u_2) = \mathrm{d}(u_3) = 2$. For a vertex $v \in N_3(u_1) \setminus \{u_4\}$, we have v is adjacent to u_4, or otherwise we have $\mathrm{d}(v, u_2) > 3$, which is a contradiction. Thus, u_4 is adjacent to all members of $N_3(u_1) \setminus \{u_4\}$ and the induced subgraph on $N_3(u_1)$ is a star graph. Since $\mathrm{d}(u_1) \geq 3$, we have $|N_2(u_1) \setminus \{u_3\}| > 1$. Assume that w_1 and w_2 are two distinct elements of $N_2(u_1) \setminus \{u_3\}$. We know that

$\mathrm{d}(w_1, w_2) \leq 3$. While $\mathrm{d}(w_1, w_2)$ equaling one or two is not possible because of the girth constraint, we must have $\mathrm{d}(w_1, w_2) = 3$. Therefore, two vertices of $N_3(u_1)$ are adjacent and form a triangle with u_4, which is a contradiction.
Case 2. There are no two adjacent vertices of degree two. Suppose that $\mathrm{d}(u) = 2$ and that $v, v' \in N_1(u)$. Since $\mathrm{d}(v) \geq 3$, there are two vertices $w, z \in N_1(v) \cap N_2(u)$. Now, consider $s \in N_1(w) \cap N_3(u)$ and $t \in N_1(z) \cap N_3(u)$. Since $N_2(v) \cap N_3(u)$ and $N_2(v') \cap N_3(u)$ are independent sets and each vertex of these sets has at most one neighbor in other set, we deduce that $\mathrm{d}(s, t) \leq 2$. If $\mathrm{d}(s, t)$ is equal to one or two, then G has a cycle of length five or six, respectively, which is a contradiction. □

Now, we show that every 3-sc graph with girth 7 is regular.

Theorem 3.3. *Let G be a 3-sc graph and girth$(G) = 7$. Then G is regular.*

Proof. If $\delta(G) = 2$, then Theorem 3.2 implies that G is isomorphic to C_7 and the assertion holds. Assume that $\delta(G) \geq 3$ and that $u \in V(G)$. Then $N_1(u)$ and $N_2(u)$ are two independent sets. Suppose that $w_1 \in N_1(u)$. Since the girth G is seven, there exists $w_2 \in N_2(u)$ such that w_1 is not adjacent to w_2. If $\mathrm{d}(w_1) \neq \mathrm{d}(w_2)$, then consider the following two cases:
Case 1. $\mathrm{d}(w_1) > \mathrm{d}(w_2)$. For each $v \in N_1(w_1) \cap N_2(u)$, we have $\mathrm{d}(v, w_2) = 3$. Otherwise, we have a cycle of length less than seven. Thus, there is a vertex in $z \in N_1(w_2) \cap N_3(u)$ such that it has two neighbors in $N_2(w_1) \cap N_3(u)$. If z is adjacent to two vertices of neighbors of v, then we have a cycle of length four. Otherwise, we have a cycle of length six. In both cases, we get a contradiction.
Case 2. $\mathrm{d}(w_1) < \mathrm{d}(w_2)$. For each $v \in N_1(w_2) \cap N_3(u)$, we have $\mathrm{d}(v, w_1) = 3$. Because if $\mathrm{d}(v, w_1)$ is equal to one or two, then we have a cycle of length five or six, respectively. Thus, v is adjacent to at least one of vertices of $N_2(w_1) \cap N_3(u)$. Since $\mathrm{d}(w_1) < \mathrm{d}(w_2)$, there exists a vertex $w \in N_1(w_1) \cap N_2(u)$ such that $|N_2(w) \cap N_1(w_2)| \geq 2$. Thus, we have a cycle of length less than seven, which is a contradiction. Thus, all vertices of $N_1(u) \cup N_2(u)$ have the same degree. Now, choose two arbitrary vertices $s, t \in V(G)$. Suppose that $\mathrm{d}(s, t) = 1$ and select $u \in N(s)$ such that $u \neq t$. In this case, $s, t \in N_1(u) \cup N_2(u)$ and have the same degree. If $\mathrm{d}(s, t) = 2$, then select $u \in N(s) \cap N(t)$. Similarly, $s, t \in N_1(u) \cup N_2(u)$ and have the same degree. Finally, if $\mathrm{d}(s, t) = 3$, then there exist vertices $a, b \in V(G)$ such that $s \sim a \sim b \sim t$. It is sufficient to consider $u = a$ and the assertion holds. □

Suppose that G is a 3-sc graph of girth seven. Theorem 3.3 implies that G is r-regular. If $u \in V(G)$, then $|N_1(u)| = r$, $|N_2(u)| = r(r-1)$, and $|N_3(u)| = r(r-1)^2$. Thus, for $r \geq 3$, we have

$$\begin{aligned} |V(G)| &= 1 + r + r(r-1) + r(r-1)^2 \\ &= \frac{r(r-1)^3 - 2}{r-2}. \end{aligned}$$

The value above is a lower bound for the number of vertices of an $(r, 7)$-cage graph [2, Exercise 3.1.12]. Therefore, for $r \in \{3, 4, 5, 6\}$, the number of vertices of $(r, 7)$-cage graph is greater than $\frac{r(r-1)^3-2}{r-2}$. Hence, there is no an r-regular 3-sc graph with girth seven, when $r \in \{3, 4, 5, 6\}$.

Problem 3.4. Is there an r-regular 3-sc graph with degree at least three such that its girth is seven?

It should be mentioned that girth$(G) = 7$ in Theorem 3.3 is not redundant because the graph of Fig. 5. is a 3-sc graph of girth six, but is not regular.

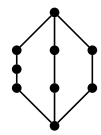

FIGURE 5. A 3-sc graph of girth six that is not regular.

Next, we state a proposition about 3-sc graphs of girth six.

Proposition 3.5. *Let G be a graph of girth six. Then G is a 3-sc graph if and only if every two vertices of G are on a cycle of length six or seven.*

Proof. Suppose that G is a 3-sc graph and that $u, v \in V(G)$. Clearly, u and v are on a cycle, and we may assume that C is the smallest cycle that contains u and v. If the length of cycle is greater than seven, then G is not a 3-sc graph. Since the girth of G is six, we deduce that the length of cycle is either six or seven. □

Next, we find the upper and lower bounds for the number of vertices, for 3-sc graphs of girth six.

Proposition 3.6. *If G is an r-regular 3-sc graph with girth six, then $2r^2-2r+2 \leq |V(G)| \leq r^3 - r^2 + r + 1$.*

Proof. Suppose that G is an r-regular 3-sc graph and that $u \in V(G)$. Clearly, $|N_1(u)| = r$ and $|N_2(u)| = r(r-1)$. The maximum number of vertices of $N_3(u)$ is $r(r-1)^2$. Suppose that $v \in N_1(u)$. Since girth$(G) = 6$, we deduce that $|N_3(u) \cap N_2(v)| = (r-1)^2$. Hence, the minimum number of $|N_3(u)|$ is $(r-1)^2$. So, the number of vertices G in this case equals $2r^2 - 2r + 2$. □

Next, we show that the lower bound is sharp.

Theorem 3.7. *There is an r-regular 3-sc graph G of girth six such that $|V(G)| = 2r^2 - 2r + 2$.*

Proof. If $u \in V(G)$ and $|V(G)| = 2r^2 - 2r + 2$, then Proposition 3.6 implies that $|N_1(u)| = r$, $|N_2(u)| = r(r-1)$, and $|N_3(u)| = (r-1)^2$. For each vertex $v \in N_1(u)$, we have $|N_3(u) \cap N_2(v)| = (r-1)^2$. Since girth$(G) = 6$, each vertex of $N_3(u)$ has just one neighbor in $N_1(v)$. From the fact that the number of v's is equal to r, the neighbors of each vertex of $N_3(u)$ are fixed. □

Now, we build a graph such that it satisfies in Theorem 3.7. Assume that u is an arbitrary vertex of G such that $N_1(u) = \{v_1, \ldots, v_r\}$ and $N_1(v_i) \setminus \{u\} = \{v_{i2}, \ldots, v_{ir}\}$, for each $1 \leq i \leq r$. Let $w_{ij}, 2 \leq i, j \leq r$ be elements of $N_3(u)$. Then, Theorem 3.7 implies that the vertices of each part $N_i(u), 1 \leq i \leq 3$ are independent. It remains just to put edges between $N_3(u)$ and $N_2(u)$. Now, two vertices w_{st} and v_{kh} are adjacent if and only if $t = h$. In this case, G satisfies Theorem 3.7.

Example 3.8. Consider the 3-regular 3-sc graph Fig. 6. Here, $\text{girth}(G) = 6$, $r = 3$, and $|V(G)| = 14$ which satisfies the formula of Theorem 3.7.

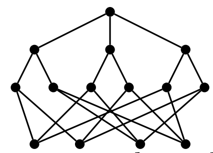

FIGURE 6. A 3-regular 3-sc graph with girth six and minimum number of vertices.

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[1]DEPARTMENT OF PURE MATHEMATICS, FERDOWSI UNIVERSITY OF MASHHAD, MASHHAD, P. O. BOX 1159, MASHHAD 91775, IRAN.

Email address: `elhamimani@mail.um.ac.ir; mirzavaziri@gmail.com`