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LOCAL SUBSPACE TRANSITIVITY CRITERION

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Abstract. An operator T on a Banach space X is called transitive, if for every nonempty open subsets U,V of X, there is a positive integer n, such that $T^n(U) \cap V \neq \phi$. In the present paper, we introduce local subspace transitivity and investigate some basic properties of local subspace transitive operators. Also, we construct some nontrivial examples and state the local subspace transitivity criterion. Then we give an example of a local subspace transitive operator which is not a topological transitive operator.

1. Introduction

Assume that X is a Banach space and $\mathcal{B}(X)$ the Banach algebra of all bounded linear operators on X. Suppose $T \in B(X)$. If for every pair U, V of nonempty open subsets of X, there is a positive integer n so that subset $T^n(U) \cap V$ is nonempty, then the operator T is transitive. If the underlying space is considered as a separable Banach space, then transitivity is equivalent to hypercyclicity. To be more clear, if B is a subset of X, then the orbit of B under an operator T is the set $orb(T, B) = \{T^n x;\ x \in B, n = 0, 1, 2, \cdots\}$. If B is a singleton $\{x\}$ and $\overline{orb(T, B)} = X$, then T is called a hypercyclic operator and x is a hypercyclic vector for T. If $B = \{\lambda x;\ \lambda \in \mathbb{C}\}$ for some vector $x \in X$ and $orb(T, B)$ is a dense subset of X, then this operator is said to be a supercyclic operator and the vector x is a supercyclic vector for T.

Birkhoff [8] introduced a topological transitive operator and gave an example which plays an important role in approximating any entire function in $H(\mathbb{C})$. Other examples can be seen in [14] and [23]. We recall that there is no hypercyclic operator on a finite-dimensional Banach space. On the other side, Ansari [1]

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showed that every infinite-dimensional separable Banach space admits a hypercyclic operator.

The concept of M-hypercyclicity was proposed in [15]. For a subspace M of X, an operator $T \in B(X)$ is M-hypercyclic (subspace-hypercyclic) if the $M \cap orb(T, x)$ is dense in M for some $x \in X$. The same authors defined the concept of subspace-transitivity. Later, Le [13] showed that M-transitivity property is stronger than M-hypercyclicity. The reader can see more information about subspace-hypercyclicity in [3, 5, 18–20] and also in [6, 12].

For a vector $x \in X$ and an operator $T \in B(X)$, $J(x)$ is defined as follows

$$J(x) = \{z \in X; \text{there exist a sequence } \{z_n\} \subset X \text{ and a strictly increasing sequence of positive integers } \{m_n\}, \text{ such that } z_n \longrightarrow x \text{ and } T^{m_n} z_n \longrightarrow z\},$$

(see [10, 11]). The same authors claimed that $J(x)$ coincides with the set

$$A := \{z \in X; \text{for every pair of neighborhoods } U,\ V \text{ of } x,\ z \text{ respectively, there exists a positive integer } n \text{ such that } T^n U \cap V \neq \phi\}.$$

In fact, they tried to localize the concept of hypercyclicity. In [2], via providing an example, it was shown that the above localization is not valid, and $J(x)$ was characterized through the open sets. In order to convince, we give a simple example. Let $T = \frac{1}{2}B$, where B is the backward shift operator on $\ell^2(\mathbb{N})$, and consider $z \in \ell^2(\mathbb{N})$ such that $Tz \neq 0$. Obviously the vector Tz belongs to A, but for every strictly increasing sequence of positive integers $\{k_n\}$ and every sequence $\{z_n\} \subset X$, if $z_n \longrightarrow z$ then $T^{k_n} z_n \longrightarrow 0$ and we get $J(x) = \{0\}$.
More information on the J-class operators can be seen in [4], [16] and [21].

In this paper, we will present the definition of M-extended limit set for a vector $x \in X$ and the local M-transitivity. Moreover, a sufficient condition is presented as subspace $\mathbb{J}_M$-class criterion for an operator $T \in B(X)$ to be the subspace $\mathbb{J}_M$-class operator.

In the following, X, $B(X)$ and M instead of a separable Banach space, the set of all linear operators on the space X and a nontrivial subspace of X, respectively. As usual, the open set in norm-topology on X, which contains a vector $x \in X$, will be shown by U_x.

2. PRELIMINARIES

Definition 2.1. An operator $T \in B(X)$ is called M-transitive if for each pair of nonempty relatively open subsets V and U of M there exists an integer $n \geq 0$, such that $V \cap T^{-n}(U)$ is nonempty and M is an invariant subspace for T^n.

Definition 2.2. For a subspace M of X, the M-extended limit set for a vector x under an operator $T \in B(X)$ is the set of all $y \in M$ such that there is a sequence $\{z_n\}$ in M and a strictly increasing sequence $\{k_n\} \subset \mathbb{N}$ such that $z_n \longrightarrow x$ and $T^{k_n} z_n \longrightarrow y$, and $T^{k_n}(M) \subseteq M$ for all $n \in \mathbb{N}$.

The M-extended limit set of a vector x under an operator T is denoted by $\mathbb{J}_M(x)$.

Remark 2.3. Let $T \in B(X)$ be an M-transitive operator and U, V nonempty relatively open subsets of M. Let $n_0 \geq 0$ be such that $T^{-n_0}(U) \cap V \neq \emptyset$ and $T^{n_0}(M) \subseteq M$. Then there exist two distinct vectors $x, y \in T^{-n_0}(U) \cap V$ and relatively open subsets O_x and O_y of M, such that

$$O_x \cap O_y = \emptyset \quad , \quad O_x, O_y \subset T^{-n_0}(U) \cap V.$$

According to the assumptions, there is an integer $k \geq 1$, such that $T^k(M) \subseteq M$ and $T^{-k}(O_x) \cap O_y \neq \emptyset$. Thus, the set $V \cap T^{-(k+n_0)}(U)$ is nonempty and, clearly, $T^{(n_0+k)}(M) \subseteq M$. By repeating the above method, it can be ensured that there is an infinite subset P of $\mathbb{N}$, such that for every $n \in P$, $V \cap T^{-n}(U) \neq \emptyset$ and $T^n(M) \subseteq M$.

Example 2.4. Let $T = \lambda B \oplus I$ in $\mathcal{B}(\ell^2(\mathbb{N}) \oplus \ell^2(\mathbb{N}))$, where $\lambda \in \mathbb{C}$ with $|\lambda| > 1$ and B is the backward shift on $\ell^2(\mathbb{N})$, i.e. for every $x = (x_0, x_1, x_2, ...) \in \ell^2(\mathbb{N})$;

$$B(x_0, x_1, x_2, ...) = (x_1, x_2, ...).$$

Clearly, the operator T is not hypercyclic. Now consider $M = \ell^2(\mathbb{N}) \oplus (0)$, then T is an M-hypercyclic operator. In fact,

$$(\lambda B)^n(x_0, x_1, x_2, ...) = \lambda^n(x_n, x_{n+1}, ...),$$

and if $U \subseteq \ell^2(\mathbb{N})$, $V \subseteq \ell^2(\mathbb{N})$, both relatively open, and $x = (x_0, x_1, x_2, ...) \in U$, $y = (y_0, y_1, y_2, ...) \in V$, then $\widehat{U} := U \oplus (0)$ and $\widehat{V} = V \oplus (0)$ are two open subsets of M. For every $n \in \mathbb{N}$ set;

$$z_n = (x_0, x_1, ..., x_{n-1}, \frac{y_0}{\lambda^n}, \frac{y_1}{\lambda^n}, ..., \frac{y_{n-1}}{\lambda^n}, \frac{y_0}{\lambda^{2n}}, \frac{y_1}{\lambda^{2n}}, ..., \frac{y_{n-1}}{\lambda^{2n}}, ...),$$

thus $z_n \in \ell^2(\mathbb{N})$, $z_n \longrightarrow x$ and $T^n z_n \longrightarrow y$ as $n \longrightarrow \infty$. Therefore there exists a non-negative integer N such that;

$$\forall n \geq N, \quad T^n(\widehat{U}) \cap \widehat{V} \neq \emptyset,$$

and obviously $T(M) \subseteq M$.

Note that, when T is an M-transitive operator and U, V are nonempty relatively open subsets of M, then the recent remark implies that there exists an infinite subset P of $\mathbb{N}$ such that for every $n \in P$ the intersection $V \cap T^{-n}(U)$ is nonempty, and this help us to give a characterization of $\mathbb{J}_M(x)$ through of relatively open subsets in the following proposition.

For an element $x \in X$ and $r > 0$, $B(x, r)$ denotes the open ball of radius r centered at x.

Proposition 2.5. *Assume that $T \in \mathcal{B}(X)$ and $x \in M$. Then $\mathbb{J}_M^{tra}(x) = \mathbb{J}_M(x)$, where $\mathbb{J}_M^{tra}(x)$ is the set of all $y \in M$ such that for every relatively open neighborhoods U_x, V_y of vectors x, y in M, respectively, and every positive integer n_0, there exists an integer $n > n_0$ such that $T^n(U_x) \cap V_y \neq \emptyset$ and $T^n(M) \subseteq M$.*

Proof. Assume that $y \in \mathbb{J}^{tra}_M(x)$, $k_0 = 1$, $N = k_{n-1}$ and for every $n \in \mathbb{N}$ set:

$$U_{(x,n)} = B(x, \frac{1}{n}) \cap M, \qquad V_{(y,n)} = B(y, \frac{1}{n}) \cap M.$$

By the assumption, an integer $k_n > N$ and a vector $x_n \in U_{(x,n)}$ exist, so that $T^{k_n}(M) \subseteq M$ and $T^{k_n}x_n \in V_{(y,n)}$. By repeating this method, we find two sequences $\{k_n\} \subset \mathbb{N}$ and $\{x_n\} \subset X$, such that $x_n \longrightarrow x$ and $T^{k_n}x_n \longrightarrow y$. This implies the nontrivial side of the proof. □

If for an operator T on X we have $\mathbb{J}_M(x) = M$ for some vector $x \in M$, then the vector x is called a subspace $\mathbb{J}$-class vector for subspace M under operator T and in this case, the operator T is called a $\mathbb{J}_M$-class operator.

According to the above proposition, an operator T on X is M-transitive if and only if $\mathbb{J}_M(x) = M$ for all $x \in M$. In the next section we will give an example of a non topological transitive operator which has a subspace $\mathbb{J}$-class vector for a certain subspace M of X.

Below, we investigate some topological properties of $\mathbb{J}_M(x)$.

Lemma 2.6. *Assume that $T \in \mathcal{B}(X)$ and two sequences $\{x_n\}$ and $\{y_n\}$ are in M such that $x_n \longrightarrow x$, $y_n \longrightarrow y$ and for all $n \geq 1$, $y_n \in \mathbb{J}_M(x_n)$, then $y \in \mathbb{J}_M(x)$.*

Proof. Suppose that N is an arbitrary positive integer and U_x and U_y are two relatively open sets in M. Then we can find an integer $n_1 > N$ and two vectors $x_{n_1} \in U_x$ and $y_{n_1} \in U_y$ with two relatively open sets $U_{x_{n_1}}$ *and* $U_{y_{n_1}}$, by Proposition 2.5, such that $U_{y_{n_1}} \subset U_y$, $U_{x_{n_1}} \subset U_x$ and $y_{n_1} \in \mathbb{J}_M(x_{n_1})$. Again by Proposition 2.5 we can find an integer $n \geq n_1$, such that

$$\emptyset \neq T^n\left(U_{x_{n_1}}\right) \cap U_{y_{n_1}} \subseteq T^n(U_x) \cap U_y$$

and

$$T^n(M) \subseteq M,$$

so the proof is completed. □

Obviously, the set $\mathbb{J}_M(x)$ is T-invariant and the closeness of the set $\mathbb{J}_M(x)$ is an immediate result of the previous lemma. Some topological properties of the set of subspace $\mathbb{J}_M$-class vectors is given in the following theorem.

Theorem 2.7. *Let $T \in \mathcal{B}(X)$ and M be a nontrivial subspace of X. Then the set of subspace $\mathbb{J}_M$-class vectors under T is a closed, connected and T-invariant set.*

Proof. Set:

$$J = \{x \in M;\ \ \mathbb{J}_M(x) = M\}.$$

Clearly, J is a T-invariant set and also, it is a closed set by Lemma 2.6. Now, assume that $x \in J$, $y \in M$, $\lambda \in \mathbb{C}$ and $\lambda \neq 0$. Then, there is a strictly increasing sequence $\{k_n\} \subset \mathbb{N}$ and a sequence $\{x_n\} \subset M$, such that;

$$x_n \longrightarrow x, \qquad T^{k_n}x_n \longrightarrow \frac{1}{\lambda}y.$$

If $\{\lambda_n\} \subset \mathbb{C}$ such that $\lambda_n \longrightarrow \lambda$, then,

$$\lambda_n x_n \longrightarrow \lambda x, \qquad T^{k_n}(\lambda_n x_n) \longrightarrow y,$$

thus $y \in \mathbb{J}_M(\lambda x)$, or equivalently, $\lambda x \in J$.
Now since the set $\mathbb{C}x$ is connected for every vector x, so J is the union of the connected sets, i.e., $J = \bigcup_{x\in J} \mathbb{C}x$. Note that, If $\lambda_n \longrightarrow 0$, then $\mathbb{J}_M(0) = M$ or $0 \in J$ and $0 \in \bigcap_{x\in J} \mathbb{C}x$, so J is a connected set. □

Proposition 2.8. *Assume that $T \in \mathcal{B}(X)$ and M is a subspace of X, $\{k_n\}$ is a fixed strictly increasing sequence of positive integers and Y is a dense subset of M such that for every $y \in Y$ there is a sequence $\{y_n\} \subset Y$ for which $y_n \longrightarrow 0$ and $T^{k_n} y_n \longrightarrow y$. In this case, for each $x \in M$ there is a sequence $\{x_n\} \subset M$ such that $x_n \longrightarrow 0$ and $T^{k_n} x_n \longrightarrow x$.*

Proof. According to the assumptions, if we fix a vector x in $M \setminus Y$, then there is a sequence $\{y_m\} \subset Y$ such that $y_m \longrightarrow x$. Note that, for each m there exists a sequence $\{w'_{m_n}\} \subset M$ with $w'_{m_n} \longrightarrow 0$ and $T^{k_n} w'_{m_n} \longrightarrow y_m$ as $n \longrightarrow \infty$. Now, we can consider the smallest number m_1 such that $y_{m_1} \in B(x, \frac{1}{2}) \cap M$. Again by the assumptions, there exists a sequence $\{w'_{m_1,n}\} \subset M$ with $w'_{m_1,n} \longrightarrow 0$ and $T^{k_n} w'_{m_1,n} \longrightarrow y_{m_1}$, as $n \longrightarrow \infty$. Hence

$$w'_{m_1,n} \in B(0, \frac{1}{2}) \cap M$$

and

$$T^{k_n} w'_{m_1,n} \in B(x, \frac{1}{2}) \cap M$$

for some integer $n_1 > 0$. so far the proof process, we set

$$x_1 = x_2 = \cdots = x_{n_1 - 1} = 0, \quad \textit{and} \quad x_{n_1} = w'_{m_1,n}.$$

Then we consider the open ball $B(x, \frac{1}{2^l})$ and the smallest integer $m_l > m_{l-1}$ such that the open ball $B(x, \frac{1}{2^l})$ contains y_{m_l}, so there is a sequence $\{w'_{m_l,n}\} \subset M$ with $w'_{m_l,n} \longrightarrow 0$ and $T^{k_n} w'_{m_l,n} \longrightarrow y_{m_l}$. Therefore, there exists an integer $n_l > n_{l-1}$ such that

$$w'_{m_l,n_l} \in B(0, \frac{1}{2^l}) \cap M \quad \textit{and} \quad T^{k_{n_l}} w'_{m_l,n_l} \in B(x, \frac{1}{2^l}) \cap M.$$

To complete the proof process, if we set

$$x_{n_{l-1}+1} = \cdots = x_{n_l - 1} = 0 \;\; \textit{and} \;\; x_{n_l} = w'_{m_l,n_l},$$

then we have the sequence $\{x_n\} \subset M$ such that $x_n \longrightarrow 0$ and $T^{k_n} x_n \longrightarrow x$, which is the desired result. □

Obviously, if Y is a dense subset of M and $Y \subset \mathbb{J}_M(0)$, then by Theorem 2.7 we have that $\mathbb{J}_M(0) = M$. But note that, this matter is different from the previous proposition, because in the previous proposition, the sequence $\{k_n\}$ was a fixed strictly increasing sequence from positive integers.

3. Local Subspace Transitivity Criterion

As hypercyclicity criterion is a useful tool for identifying hypercyclic operators and provides a sufficient condition to ensure that an operator is hypercyclic, we also intend to introduce the local subspace transitivity criterion. Also, we will provide an example that satisfies this criterion. Some useful references containing examples and properties of hypercyclicity, subspace hypercyclicity and transitivity criterions are [7, 9, 17, 25, 26].

Theorem 3.1 (**Local Subspace Transitivity Criterion**). *Let M be a subspace of a Banach space X and $T \in \mathcal{B}(X)$. If Y is a dense subset of M and for some vector $x \in M$ there exists a strictly increasing sequence $\{k_n\} \subset \mathbb{N}$ such that*

(1) *There is a sequence $\{z_n\} \subset M$, such that $z_n \longrightarrow x$ and $T^{k_n} z_n \longrightarrow 0$,*
(2) *For every $y \in Y$, there is a sequence $\{w_n\} \subset M$, such that $w_n \longrightarrow 0$ and $T^{k_n} w_n \longrightarrow y$,*
(3) *For every $n \in \mathbb{N}$, M is an invariant subspace under T^{k_n},*

then x is a subspace $\mathbb{J}$-class vector for M under T.

Proof. Assume that $y \in M$ and integer $N \geq 1$; also, for sufficiently small $\varepsilon > 0$, $W_1' := B(0, \frac{\varepsilon}{2})$ is the relatively open ball in M, and we set

$$U_x := W_1' + x, \qquad U_y := W_1' + y.$$

By the assumptions, there is an integer $k_1 \geq N$ such that

$$T^{k_1}(U_x) \cap W_1' \neq \emptyset \quad \textit{and} \quad T^{k_1}(W_1') \cap U_y \neq \emptyset.$$

So there are vectors $w_1, w_2, w_3, w_4 \in W_1'$ such that

$$T^{k_1}(w_1 + x) = w_2, \qquad T^{k_1}(w_3) = w_4 + y.$$

Hence

$$T^{k_1}(x + w_1 + w_3) = w_2 + w_4 + y$$

and

$$x_1 := x + w_1 + w_3 \in B(x, \varepsilon), \qquad y_1 := y + w_2 + w_4 \in B(y, \varepsilon).$$

Similarly likewise, we can construct the sequences $\{x_n\} \subset M$, $\{y_n\} \subset M$ and $\{k_n\} \subset \mathbb{N}$ such that for each $n \geq 2$ we have that $k_n - 1 > k_{n-1}$,

$$x_n \in B(x, \frac{\varepsilon}{n}), \qquad y_n \in B(y, \frac{\varepsilon}{n})$$

and $T^{k_n} x_n \longrightarrow y$ as $n \longrightarrow \infty$. Therefore the assertion (3) shows that x is a subspace $\mathbb{J}$-class vector for subspace M under T. □

In the following, we give an example of an operator which is not a topological transitive operator but satisfies the local subspace transitivity criterion.

It is worthwhile to mention that, the unilateral weighted backward shift T on $\ell^2(\mathbb{N})$ with the weight sequence $\{w_n\}_{n\in\mathbb{N}} \subseteq \mathbb{N}$ is topological transitive if and only if $\limsup_n(\Pi_{i=1}^n w_i) = +\infty$, [23]. Consequently, the operator which arises in the next example is not a topological transitive operator.

Example 3.2. Consider the weighted backward shift operator T on $\ell^2(\mathbb{N})$ given by

$$T(x^1, x^2, \cdots) = (2x^2, \frac{3}{2}x^3, \frac{4}{3}x^4, \cdots).$$

Also let $Y^{'}$ be the set of all finite sequences with entries $z \in \mathbb{C}$ that $Re(z) \in \mathbb{Q}$, $Im(z) \in \mathbb{Q}$, and let M be the following subspace of $\ell^2(\mathbb{N})$:

$$M = \{\{x_n\} \in \ell^2(\mathbb{N})\ ;\ x_{2k} = 0,\ for\ all\ k \in \mathbb{N}\}.$$

Since $Y := Y^{'} \cap M$ is dense in M, so there are a strictly increasing sequence $\{2k\}_k$ with a sequence $\{x_k\} \subset Y$ such that

$$x_k = (x^1, 0, x^3, 0, \cdots, x^{2k-1}, 0, 0, \cdots),$$

$x_k \longrightarrow 0$ as $k \longrightarrow \infty$ and $T^{2k}x_k = 0$. Now, for an arbitrary member of Y such as $y = (y^1, 0, y^3, 0, \cdots, y^{2m+1}, 0, 0, \cdots)$ and $k \geq 1$, we set

$$w_{2k}(y) = (\underbrace{0, \cdots, 0}_{2k-times}, \frac{y^1}{2k+1}, 0\ , \frac{3y^3}{2k+3},\ 0, \cdots, \frac{(2m+1)y^{2m+1}}{2(k+m)+1},\ 0,\ 0, \cdots).$$

Clearly, the sequence $\{w_{2k}(y)\}$ is a sequence in Y, and $w_{2k}(y) \longrightarrow 0$, as $k \longrightarrow \infty$, because

$$||w_{2k}(y)||^2 = \sum_{j=1}^{2m} |\frac{j}{2k+j}y^j|^2 \leq \frac{4m^2}{(2k+1)^2}||y||^2.$$

Moreover, since for $n \geq 1$ we have that

$$T^n(x^1, x^2, x^3, \cdots) = \left((n+1)x^{n+1},\ \frac{1}{2}(n+2)x^{n+2},\ \frac{1}{3}(n+3)x^{n+3},\ \cdots\right),$$

so

$$T^{2k}w_{2k}(y) = \left((2k+1)\frac{1}{2k+1}y^1, 0, (\frac{2k+3}{3})(\frac{3}{2k+3})y^3, 0, \cdots, \right.$$
$$\left. (\frac{2(k+m)+1}{(2m+1)})(\frac{2m+1}{2(k+m)+1})y^{2m+1}, 0, 0, \cdots\right) = y,$$

or in other words, the condition (2) holds. Therefore, the operator T satisfies the local subspace transitivity criterion with respect to M and $\mathbb{J}_M(0) = M$, because the condition (3) clearly holds.

Now we come to the end of the paper with some questions on the local subspace $\mathbb{J}$-class operators.

It is well known that if an operator T is hypercyclic, then any power of T is a hyperclic operator [**?**]. So the first question arises as follows.

Problem 3.3. Let X be a Banach space and $T \in \mathcal{B}(X)$. If T is a $\mathbb{J}_M$-class operator with respect to some subspace M, is T^n also a subspace $\mathbb{J}_M$-class operator operator for every $n > 1$? If so, what is the relation between their subspace $\mathbb{J}$-class vectors?

The first example of a topologically transitive operator whose adjoint is also a topologically transitive operator was presented by Salas, [22]. The same author showed that every separable Banach space with separable dual space, supports such an operator, [24]. Now we can ask the following question.

Problem 3.4. Let X be a Banach space. Dose there exist a non-topologically transitive operator $T \in \mathcal{B}(X)$ such that both T^* and T are subspace $\mathbb{J}$-class operators?

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