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FEKETE-SZEGÖ INEQUALITIES FOR CERTAIN SUBCLASSES OF STARLIKE AND CONVEX FUNCTIONS OF COMPLEX ORDER ASSOCIATED WITH QUASI-SUBORDINATION

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Abstract. In this paper, we find Fekete-Szegö bounds for a generalized class $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$. Also, we discuss some remarkable results.

1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. Further, by $\mathcal{S}$ we shall denote the class of all functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$.

For two functions f and g, analytic in $\mathbb{U}$, we say that the function $f(z)$ is subordinate to $g(z)$ in $\mathbb{U}$, and write

$$f(z) \prec g(z) \qquad (z \in \mathbb{U})$$

if there exists a Schwarz function $w(z)$, analytic in $\mathbb{U}$, with

$$w(0) = 0 \quad \text{and} \quad |w(z)| < 1 \qquad (z \in \mathbb{U})$$

such that

$$f(z) = g(w(z)) \qquad (z \in \mathbb{U})\,.$$

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In particular, if the function g is univalent in $\mathbb{U}$, the above subordination is equivalent to

$$f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

An analytic function $f(z)$ is quasi-subordinate to an analytic function $g(z)$ in the open unit disc $\mathbb{U}$ if there exist analytic function h with $|h(z)| \leq 1$, such that $\frac{f(z)}{h(z)}$ is analytic in $\mathbb{U}$ and

$$\frac{f(z)}{h(z)} \prec g(z) \qquad (z \in \mathbb{U}).$$

We also denote the above expression by

$$f(z) \prec_q g(z) \qquad (z \in \mathbb{U})$$

and this is equivalent to

$$f(z) = h(z)g(w(z)) \qquad (z \in \mathbb{U})$$

where w, is analytic with $w(0) = 0$ and $|w(z)| < 1$.

If $h(z) \equiv 1$, then $f(z) = g(w(z))$, which implies that $f(z) \prec g(z)$ in $\mathbb{U}$. Further, if $w(z) = z$, then $f(z) = h(z)g(z)$ and denoted by $f(z) \ll g(z)$ in $\mathbb{U}$ (see [3, 13, 14]).

Let $\varphi(z)$ be an analytic function with positive real part on $\mathbb{U}$ with $\varphi(0) = 1$, $\varphi'(0) > 0$ which maps the unit disk $\mathbb{U}$ onto the region starlike with respect to 1, $\varphi(\mathbb{U})$ is symmetric with respect to the real axis. The Taylor's series expansion of such function is

$$\varphi(z) = 1 + \phi_1 z + \phi_2 z^2 + \phi_3 z^3 + \dots, \tag{1.2}$$

where all coefficients are real and $\phi_1 > 0$.

Recently, El-Ashwah and Kanas [5] introduced and studied the following two subclasses:

$$\mathcal{S}_q^*(\gamma, \varphi) := \left\{ f : f \in \mathcal{A} \quad \text{and} \quad \frac{1}{\gamma}\left(\frac{zf'(z)}{f(z)} - 1\right) \prec_q \varphi(z) - 1; \quad z \in \mathbb{U},\, \gamma \in \mathbb{C} \setminus \{0\} \right\} \tag{1.3}$$

and

$$\mathcal{K}_q(\gamma, \varphi) := \left\{ f : f \in \mathcal{A} \text{ and } \frac{1}{\gamma}\frac{zf''(z)}{f'(z)} \prec_q \varphi(z) - 1; \quad z \in \mathbb{U},\, \gamma \in \mathbb{C} \setminus \{0\} \right\}. \tag{1.4}$$

We note that, when $h(z) \equiv 1$, the classes $\mathcal{S}_q^*(\gamma, \varphi)$ and $\mathcal{K}_q(\gamma, \varphi)$ reduce respectively, to the familiar classes $\mathcal{S}^*(\gamma, \varphi)$ and $\mathcal{K}(\gamma, \varphi)$ of Ma-Minda starlike and convex functions of complex order γ ($\gamma \in \mathbb{C}\backslash\{0\}$) in $\mathbb{U}$ (see [12]). For $\gamma = 1$, the classes $\mathcal{S}^*(\gamma, \varphi)$ and $\mathcal{K}(\gamma, \varphi)$ reduce respectively to the unified classes $\mathcal{S}^*(\varphi)$ and $\mathcal{K}(\varphi)$ of starlike and convex functions of Ma-Minda type (see [10]). For $\gamma = 1$, the classes $\mathcal{S}_q^*(\gamma, \varphi)$ and $\mathcal{K}_q(\gamma, \varphi)$ reduce to the classes $\mathcal{S}_q^*(\varphi)$ and $\mathcal{K}_q(\varphi)$, respectively, introduced by Haji Mohd and Darus [8]. Further, Gurusamy et al. [7] discussed these classes $\mathcal{S}_q^*(\varphi)$ and $\mathcal{K}_q(\varphi)$ by using the $k-^{th}$ root transformation.

Motivated by the works of Haji Mohd and Darus [8], in this paper we define the following subclass:

Definition 1.1. Let the class $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$, $0\neq\gamma\in\mathbb{C}$, $\delta\geq 0$, consist of functions $f\in\mathcal{A}$ satisfying the quasi-subordination

$$\frac{1}{\gamma}\left((1-\delta)\frac{z\mathcal{H}_\lambda'(z)}{\mathcal{H}_\lambda(z)}+\delta\left(1+\frac{z\mathcal{H}_\lambda''(z)}{\mathcal{H}_\lambda'(z)}\right)-1\right)\prec_q \varphi(z)-1, \tag{1.5}$$

where

$$\mathcal{H}_\lambda(z)=(1-\lambda)f(z)+\lambda z f'(z), \quad (0\leq\lambda\leq 1).$$

Example 1.2. A function $f:\mathbb{U}\to\mathbb{C}$ defined by the following:

$$\frac{1}{\gamma}\left((1-\delta)\frac{z\mathcal{H}_\lambda'(z)}{\mathcal{H}_\lambda(z)}+\delta\left(1+\frac{z\mathcal{H}_\lambda''(z)}{\mathcal{H}_\lambda'(z)}\right)-1\right)=z(\varphi(z)-1), \tag{1.6}$$

belongs to the class $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$, $0\neq\gamma\in\mathbb{C}$, $\delta\geq 0$.

Throughout this work, we assume $\varphi(z)$ is an analytic function with $\varphi(0)=1$.

For special values of the parameters and φ, the class $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$ reduces to the following well known and new subclasses:

Remark 1.3. When $\lambda=0$ in the above class, we have $\mathcal{M}_q^{\delta,0}(\gamma,\varphi):=\mathcal{M}_q^{\delta}(\gamma,\varphi)$. For $\gamma=1$, we have $\mathcal{M}_q^{\delta}(1,\varphi):=\mathcal{M}_q^{\delta}(\varphi)$ [8, Definition 1.7]. Also, for $h(z)\equiv 1$ we get $\mathcal{M}_q^{\delta}(\varphi):=\mathcal{M}^{\delta}(\varphi)$ [2]. If

$$\varphi(z)=\frac{1+(1-2\alpha)z}{1-z} \qquad (0\leq\alpha<1) \tag{1.7}$$

in $\mathcal{M}^{\delta}(\varphi)$, we have $\mathcal{M}^{\delta}(\alpha)$, [11] and setting

$$\varphi(z)=\left(\frac{1+z}{1-z}\right)^{\beta} \qquad (0<\beta\leq 1) \tag{1.8}$$

in $\mathcal{M}^{\delta}(\varphi)$, we have $\mathcal{M}^{\delta}(\beta)$, [16].

Remark 1.4. When $\lambda=0$ and $\delta=0$ in $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$, we have $\mathcal{M}_q^{0,0}(\gamma,\varphi):=\mathcal{S}_q^{*}(\gamma,\varphi)$. For $\gamma=1$, $\mathcal{S}_q^{*}(1,\varphi):=\mathcal{S}_q^{*}(\varphi)$. For $h(z)\equiv 1$, we have $\mathcal{S}_q^{*}(\gamma,\varphi):=\mathcal{S}^{*}(\gamma,\varphi)$ [12]. Also, for $h(z)\equiv 1$, we get $\mathcal{S}_q^{*}(\varphi):=\mathcal{S}^{*}(\varphi)$. For $\varphi(z)$ given by (1.7), we have $\mathcal{S}^{*}(\alpha)$.

Remark 1.5. When $\lambda=0$ and $\delta=1$ in $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$, we get $\mathcal{M}_q^{1,0}(\gamma,\varphi):=\mathcal{K}_q(\gamma,\varphi)$. For $\gamma=1$, we get $\mathcal{K}_q(1,\varphi):=\mathcal{K}_q(\varphi)$. For $h(z)\equiv 1$, we have $\mathcal{K}_q(\gamma,\varphi):=\mathcal{K}(\gamma,\varphi)$ [12] and $\mathcal{K}_q(\varphi):=\mathcal{K}(\varphi)$. For $\varphi(z)$ given by (1.7), we have $\mathcal{K}(\alpha)$.

Remark 1.6. When $\delta=0$, we get $\mathcal{M}_q^{0,\lambda}(\gamma,\varphi)\equiv\mathcal{P}_q(\gamma,\lambda,\varphi)$. For $h(z)\equiv 1$, we get the class $\mathcal{P}_q(\gamma,\lambda,\varphi):=\mathcal{P}(\gamma,\lambda,\varphi)$ of starlike and convex functions of Pascu type class.

Remark 1.7. When $\delta=1$, we obtain $\mathcal{M}_q^{1,\lambda}(\gamma,\varphi)\equiv\mathcal{K}_q(\gamma,\lambda,\varphi)$. For $\gamma=1$, we have the class $\mathcal{K}_q(\lambda,\varphi)$ [15].

Inspired by the aforecited works and from the literatures [1, 5, 7, 6, 8, 15], in this paper we introduce an unified univalent function class $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$ as defined above and obtain the upper bounds for $|a_2|$ and $|a_3|$ for $f\in\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$. Also, we obtain $|a_3-\mu a_2^2|$. Moreover, we obtain the upper bounds for different new subclasses which are obtained from our defined unified class. To discuss main results we consider the following lemmas.

Lemma 1.8. [9] *Let w be the analytic function in $\mathbb{U}$, with $w(0) = 0$, $|w(z)| < 1$ and $w(z) = w_1 z + w_2 z^2 + \dots$, then $|w_2 - \tau w_1^2| \leq \max[1; |\tau|]$, where $\tau \in \mathbb{C}$. The result is sharp for the functions $w(z) = z^2$ or $w(z) = z$.*

Lemma 1.9. [4] *Let w be the analytic function in $\mathbb{U}$, with $w(0) = 0$, $|w(z)| < 1$ and let $w(z) = w_1 z + w_2 z^2 + \dots$. Then*

$$|w_n| \leq \begin{cases} 1, & n = 1; \\ 1 - |w_1|^2, & n \geq 2. \end{cases}$$

The result is sharp for the functions $w(z) = z^n$ or $w(z) = z$.

Lemma 1.10. [9] *Let $h(z)$ be the analytic function in $\mathbb{U}$, with $|h(z)| < 1$ and let $h(z) = h_0 + h_1 z + h_2 z^2 + \dots$. Then $|h_0| \leq 1$ and $|h_n| \leq 1 - |h_0|^2 \leq 1$, for $n > 0$.*

Let f be of the form (1.1), $\varphi(z) = 1 + \phi_1 z + \phi_2 z^2 + \phi_3 z^3 + \dots$, $h(z) = h_0 + h_1 z + h_2 z^2 + \dots$ and $w(z) = w_1 z + w_2 z^2 + \dots$, throughout this article unless otherwise mentioned.

2. Fekete-Szegö Results

Theorem 2.1. *If $f \in \mathcal{M}_q^{\delta,\lambda}(\gamma, \varphi)$. Then*

$$|a_2| \leq \frac{|\gamma|\phi_1}{(1+\delta)(1+\lambda)},$$

$$|a_3| \leq \frac{|\gamma| \left\{\phi_1 + max\left\{\phi_1, \left|\frac{\gamma(1+3\delta)}{(1+\delta)^2(1+\lambda)^2}\right| \phi_1^2 + |\phi_2|\right\}\right\}}{2(1+2\delta)(1+2\lambda)}$$

and for $\mu \in \mathbb{C}$

$$\left|a_3 - \mu a_2^2\right| \leq \frac{|\gamma| \left\{\phi_1 + max\left\{\phi_1, \left|\frac{\gamma(1+3\delta) - 2\mu\gamma(1+2\delta)(1+2\lambda)}{(1+\delta)^2(1+\lambda)^2}\right| \phi_1^2 + |\phi_2|\right\}\right\}}{2(1+2\delta)(1+2\lambda)}.$$

Proof. Let $f \in \mathcal{A}$ belongs to the class $\mathcal{M}_q^{\delta,\lambda}(\gamma, \varphi)$. Then there exist analytic functions h and w with $|h(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$ such that

$$\frac{1}{\gamma}\left((1-\delta)\frac{z\mathcal{H}_\lambda'(z)}{\mathcal{H}_\lambda(z)} + \delta\left(1 + \frac{z\mathcal{H}_\lambda''(z)}{\mathcal{H}_\lambda'(z)}\right) - 1\right) = h(z)(\varphi(w(z)) - 1) \quad (2.1)$$

and

$$h(z)\left(\varphi\left(w(z)\right) - 1\right) = h_0\phi_1 w_1 z + \left[h_1\phi_1 w_1 + h_0(\phi_1 w_2 + \phi_2 w_1^2)\right] z^2 + \cdots. \quad (2.2)$$

From equations (2.1) and (2.2) we get

$$\frac{1}{\gamma}(1+\delta)(1+\lambda)a_2 = h_0\phi_1 w_1 \quad (2.3)$$

and

$$\frac{1}{\gamma}\left[2(1+2\delta)(1+2\lambda)a_3 - (1+3\delta)(1+\lambda)^2 a_2^2\right] = h_1\phi_1 w_1 + h_0\phi_1 w_2 + h_0\phi_2 w_1^2. \quad (2.4)$$

Equation (2.3) gives

$$a_2 = \frac{\gamma h_0 \phi_1 w_1}{(1+\delta)(1+\lambda)}. \quad (2.5)$$

Subtracting equation (2.4) from equation (2.3) and applying equation (2.5) we get

$$a_3 = \frac{\gamma}{2(1+2\delta)(1+2\lambda)}\left[h_1\phi_1 w_1 + h_0\phi_1 w_2 + \left(h_0\phi_2 + \frac{\gamma h_0^2\phi_1^2(1+3\delta)}{(1+\delta)^2}\right) w_1^2\right]. \quad (2.6)$$

From the hypothesis of the definition $h(z)$ is analytic and bounded in $\mathbb{U}$. Using the fact

$$|h_n| \leq 1 - |h_0|^2 \leq 1 \qquad (n > 0),$$

and the well-known inequality (see Lemma 1.9)

$$|w_1| \leq 1.$$

we have

$$|a_2| \leq \frac{|\gamma|\phi_1}{(1+\delta)(1+\lambda)}.$$

Further, for $\mu \in \mathbb{C}$

$$\begin{aligned} a_3 - \mu a_2^2 \quad &= \tfrac{\gamma\phi_1}{2(1+2\delta)(1+2\lambda)} \Big\{ h_1 w_1 + h_0 \Big(w_2 + \Big[\tfrac{\phi_2}{\phi_1} + \tfrac{\gamma h_0 \phi_1 (1+3\delta)}{(1+\delta)^2} \\ &- \tfrac{\gamma h_0 \phi_1 (1+2\delta)(1+2\lambda)}{(1+\delta)^2(1+\lambda)^2} + \tfrac{\gamma h_0 \phi_1 (1-2\mu)(1+2\delta)(1+2\lambda)}{(1+\delta)^2(1+\lambda)^2} \Big] w_1^2 \Big) \Big\}. \end{aligned} \tag{2.7}$$

Again using the inequalities $|h_1| \leq 1$ and $|w_1| \leq 1$, we get

$$\begin{aligned} |a_3 - \mu a_2^2| \leq &\frac{|\gamma|\phi_1}{2(1+2\delta)(1+2\lambda)} \Bigg\{ 1 + \Bigg| w_2 - \Bigg[\frac{-\phi_2}{\phi_1} \\ &- \frac{\gamma(1+3\delta) - \gamma(1+2\delta)(1+2\lambda) + \gamma(1-2\mu)(1+2\delta)(1+2\lambda)}{(1+\delta)^2(1+\lambda)^2} h_0 \phi_1 \Bigg] w_1^2 \Bigg| \Bigg\}. \end{aligned}$$

In view of Lemma 1.8 we have

$$\left| a_3 - \mu a_2^2 \right| \leq \frac{|\gamma| \left\{ \phi_1 + max \left\{ \phi_1, \left| \frac{\gamma(1+3\delta) - 2\mu\gamma(1+2\delta)(1+2\lambda)}{(1+\delta)^2(1+\lambda)^2} \right| \phi_1^2 + |\phi_2| \right\} \right\}}{2(1+2\delta)(1+2\lambda)}.$$

For $\mu = 0$, we obtain

$$|a_3| \leq \frac{|\gamma| \left\{ \phi_1 + max \left\{ \phi_1, \left| \frac{\gamma(1+3\delta)}{(1+\delta)^2(1+\lambda)^2} \right| \phi_1^2 + |\phi_2| \right\} \right\}}{2(1+2\delta)(1+2\lambda)},$$

which completes the proof of Theorem 2.1. □

Theorem 2.2. *If $f \in \mathcal{A}$ satisfies*

$$\frac{1}{\gamma} \left((1-\delta) \frac{z\mathcal{H}_\lambda'(z)}{\mathcal{H}_\lambda(z)} + \delta \left(1 + \frac{z\mathcal{H}_\lambda''(z)}{\mathcal{H}_\lambda'(z)} \right) - 1 \right) \ll \varphi(w(z)) - 1, \tag{2.8}$$

then

$$|a_2| \leq \frac{|\gamma|\phi_1}{(1+\delta)(1+\lambda)},$$

$$|a_3| \leq \frac{|\gamma| \left\{ \phi_1 + \left| \frac{\gamma(1+3\delta)}{(1+\delta)^2(1+\lambda)^2} \right| \phi_1^2 + |\phi_2| \right\}}{2(1+2\delta)(1+2\lambda)},$$

and for $\mu \in \mathbb{C}$

$$\left| a_3 - \mu a_2^2 \right| \leq \frac{|\gamma| \left\{ \phi_1 + \left| \frac{\gamma(1+3\delta) - 2\mu\gamma(1+2\delta)(1+2\lambda)}{(1+\delta)^2(1+\lambda)^2} \right| \phi_1^2 + |\phi_2| \right\}}{2(1+2\delta)(1+2\lambda)}.$$

In light of Remarks 1.3 to 1.7, we have following corollaries.

Corollary 2.3. *If* $f \in \mathcal{S}_q^*(\gamma, \varphi)$, *then*

$$|a_2| \le |\gamma|\phi_1,$$

$$|a_3| \le \frac{|\gamma|}{2}\left[\phi_1 + max\left\{\phi_1, |\gamma|\phi_1^2 + |\phi_2|\right\}\right],$$

and for $\mu \in \mathbb{C}$

$$\left|a_3 - \mu a_2^2\right| \le \frac{|\gamma|}{2}\left[\phi_1 + max\left\{\phi_1, |\gamma||1-2\mu|\phi_1^2 + |\phi_2|\right\}\right].$$

Remark 2.4. For $\gamma = 1$, Corollary 2.3 reduces to [8, Theorem 2.1].

Corollary 2.5. *If* $f \in \mathcal{K}_q(\gamma, \varphi)$, *then*

$$|a_2| \le \frac{|\gamma|\phi_1}{2},$$

$$|a_3| \le \frac{|\gamma|}{6}\left[\phi_1 + max\left\{\phi_1, |\gamma|\,\phi_1^2 + |\phi_2|\right\}\right],$$

and for $\mu \in \mathbb{C}$

$$\left|a_3 - \mu a_2^2\right| \le \frac{|\gamma|}{6}\left[\phi_1 + max\left\{\phi_1, \frac{|\gamma||2-3\mu|}{2}\phi_1^2 + |\phi_2|\right\}\right].$$

Remark 2.6. For $\gamma = 1$, Corollary 2.5 reduces to [8, Theorem 2.4].

Corollary 2.7. *If* $f \in \mathcal{M}_q^{\delta}(\gamma, \varphi)$, *then*

$$|a_2| \le \frac{|\gamma|\phi_1}{1+\delta},$$

$$|a_3| \le \frac{|\gamma|}{2(1+2\delta)}\left[\phi_1 + max\left\{\phi_1, \frac{(1+3\delta)}{(1+\delta)^2}|\gamma|\phi_1^2 + |\phi_2|\right\}\right],$$

and for $\mu \in \mathbb{C}$

$$\left|a_3 - \mu a_2^2\right| \le \frac{|\gamma|}{2(1+2\delta)}\left[\phi_1 + max\left\{\phi_1, \left|\frac{(1+3\delta) - 2\mu(1+2\delta)}{(1+\delta)^2}\right||\gamma|\phi_1^2 + |\phi_2|\right\}\right].$$

Remark 2.8. For $\gamma = 1$, Corollary 2.7 reduces to [8, Theorem 2.10].

Corollary 2.9. *If* $f \in \mathcal{P}_q(\gamma, \lambda, \varphi)$, *then*

$$|a_2| \le \frac{|\gamma|\phi_1}{1+\lambda},$$

$$|a_3| \le \frac{|\gamma|}{2(1+2\lambda)}\left[\phi_1 + max\left\{\phi_1, \frac{|\gamma|\phi_1^2}{(1+\lambda)^2} + |\phi_2|\right\}\right],$$

and for $\mu \in \mathbb{C}$

$$\left|a_3 - \mu a_2^2\right| \le \frac{|\gamma|}{2(1+2\lambda)}\left[\phi_1 + \max\left\{\phi_1, \frac{|1-2\mu(1+2\lambda)|}{(1+\lambda)^2}|\gamma|\phi_1^2 + |\phi_2|\right\}\right].$$

Corollary 2.10. *If* $f \in \mathcal{K}_q(\gamma, \lambda, \varphi)$, *then*

$$|a_2| \le \frac{|\gamma|\phi_1}{2(1+\lambda)},$$

$$|a_3| \le \frac{|\gamma|}{6(1+2\lambda)}\left[\phi_1 + \max\left\{\phi_1, \frac{|\gamma|\phi_1^2}{(1+\lambda)^2} + |\phi_2|\right\}\right],$$

and for $\mu \in \mathbb{C}$

$$\left|a_3 - \mu a_2^2\right| \leq \frac{|\gamma|}{6(1+2\lambda)} \left[\phi_1 + \max\left\{\phi_1, \left|\frac{2-3\mu(1+2\lambda)}{2(1+\lambda)^2}\right| |\gamma|\phi_1^2 + |\phi_2|\right\}\right].$$

Remark 2.11. For $\gamma = 1$, Corollary 2.10 correct the results in [15, Theorem 2.1].

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