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EXPONENTIAL STABILITY ANALYSIS
FOR DELAY-DIFFERENTIAL SYSTEMS OF NEUTRAL TYPE
WITH AN LMI APPROACH

VEERAKYATHAIAH UMESHA[1*], SRIRANGAIAH PADMANABHAN [2], P. BASKAR [3]
AND MUHAMMAD SYED ALI [4]

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ABSTRACT. In this paper for neutral delay differential systems, the problem of determining the exponential stability is investigated. Based on the Lyapunov method, we present some useful criteria of exponential stability for the derived systems. The stability criterion is formulated in terms of linear matrix inequality (LMI),which can be easily solved by using the MATLAB LMI toolbox. Numerical examples are included to illustrate the proposed method.

1. INTRODUCTION

The problem of stability analysis of delay differential systems has investigated by many researchers in many papers from the last three decades because of the existence of delays in various practical problems; like as large class of electrical networks containing lossless transmission lines, long transmission lines in pneumatic, mechanics, chemical processes, turbojet engine, microwave oscillator, nuclear reactor, and also instability in many control systems problems. Therefore stability analysis has gotten more attention, and analyzing stability, by using the Lyapunov method or formation of linear matrix inequality (LMI) method, is one of the general methodology applied by many researchers and some of the works are as follows.

In [6] M. Liu utilized the concept of LMI approach analyzed Global exponential stability for neutral differential systems with time varying or constant delay and

P.-L. Liu in [7, 8, 10, 9] worked on stability of uncertain time-varying delay singular systems, exponential stability for uncertain robust time-varying delay systems with delay dependence , continuous and discrete time-delay grey systems, stability criteria for time delay singular systems with delay-dependence and also stability for linear time-delay systems with delay dependence.

In [14] and [13] Phat et al. investigated on stability and stabilization of switched linear discrete-time systems with interval time-varying delay and also extended the work on this system to exponential stability of this system.

In [16] Yu and in [15] Singh, using LMI, analyzed global asymptotic stability of neural networks with time varying delays and global robust stability of delayed neural networks.

In [4, 11, 17] authors utilized the effectiveness of linear matrix inequalities(LMI's) worked on stability criterion of time delay systems with nonlinear uncertainties, robust stability of linear neutral systems with nonlinear parameter perturbations, and stability criterion for a class of linear systems with time-varying delay and nonlinear perturbations correspondingly.

Not only limited to these researchers, many mathematicians and scientists gave the contribution for the field of stability analysis of various systems in diversified fields, using LMI's or Lyapunov method's inspiring of these works. Here we continue the work on analyses of exponential stability for the neutral delay differential systems and frame the LMI's to solve the system. MATLAB LMI toolbox is used to solve the LMI's in illustrative examples.

Notation

- In what follows $\mathbb{R}^n$ denote the n-dimensional real space.
- Also $\mathbb{R}^{n*n}$ denotes the set of all real $n*n$ matrices.
- $\lambda_M(A)$ and $\lambda_m(A)$ denote the maximal and minimal eigenvalue of A, respectively.
- $\|x\|$ denotes the Euclid norm of the vector of x.
- $\|A\|$ denotes the induced norm of the matrix A, that is, $\|A\| = \sqrt{\lambda_M(A^T A)}$.
- $h = h(t)$.
- $d = \dot{h}(t)$.

2. Problem statement and preliminaries

Consider the linear neutral differential system with time varying delays

$$\dot{x}(t) = Ax(t) + Bx(t - h(t)) + C\dot{x}(t - h(t)) \tag{2.1}$$

with the initial condition function

$$x(t_0 + \theta) = \Phi(\theta) \quad \forall \theta \in (-\rho, 0), \tag{2.2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A, B, C \in R^{(n*n)}$ are constant matrices, and $h(t)$ is a positive time-varying differentiable bounded delays satisfying

$$0 < h(t) \leq \bar{h} < \infty, \qquad \dot{h}(t) \leq 1, \tag{2.3}$$

where $\bar{h} = maxh(t)$ $\rho = max(\bar{h})$, $\phi(.)$ is the given continuously differentiable function on $(-\rho, 0)$, and the system of matrix A is assumed to be a Hurwitz

matrix. The system given in (2.1) often appears in the theory of automatic control or population dynamics. First, we establish a delay-independent criterion, for the exponential stability of the delay-differential system (2.1), using the Lyapunov method in terms of LMI.

Definition 2.1. [10]. The system (2.1) is said to be globally exponentially stable with a convergence rate α, if there are two positive constants α and λ such that

$$\|x(t)\| \leq \lambda e^{-\alpha t}, \qquad t \geq 0. \tag{2.4}$$

Lemma 2.2. [9]. *For any constant matrix $M \in R^{n*n}$, let $M = M^T \geq 0$, and consider scalar $\eta \geq 0$, and let the vector function $w : [o, \eta] \longrightarrow \mathbb{R}^n$ such that the concerned integrations are well defined. Then*

$$\left[\int_0^\eta w(s)ds \right]^T M \left[\int_0^\eta w(s)ds \right] \leq \eta \int_0^\eta w(s)ds. \tag{2.5}$$

Lemma 2.3. [1]. *The following matrix inequality*

$$\begin{bmatrix} Q(x) & s(x) \\ s^T(x) & R(x) \end{bmatrix} < 0, \tag{2.6}$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$, and $S(x)$, depend affinely on x, is equivalent to $R(x) > 0, Q(x) > 0$, and

$$Q(x) - S(x)R^{-1}(x)S^T(x) < 0, \tag{2.7}$$

This lemma is also called the Schur-complement lemma.

3. MAIN RESULT

Theorem 3.1. *The system (2.1) is globally exponentially stable with the convergence rate of α with a scalar $h > 0$, if there exist positive-definite symmetric matrices $P, Q, R, S \in R^{(n*n)}$ with appropriate dimensions such that the following LMI holds:* $\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13}, \\ \phi_{12}^T & \phi_{22} & \phi_{23}, \\ \phi_{13}^T & \phi_{23}^T & \phi_{33}, \end{bmatrix} < 0,$

where

$$\phi_{11} = A^T P + PA + 2\alpha P + Q + A^T AR + h^2 A^T AS - e^{-2\alpha h} S,$$
$$\phi_{12} = PB + A^T RB + h^2 A^T BS + e^{-2\alpha h} S,$$
$$\phi_{13} = PC + A^T RC + h^2 A^T CS,$$
$$\phi_{22} = B^T BR - (1-d)e^{-2\alpha h} Q + h^2 B^T BS - e^{-2\alpha h} S,$$
$$\phi_{23} = B^T CR + h^2 SB^T C,$$
$$\phi_{33} = C^T CR - (1-d)e^{-2\alpha h} R + h^2 C^T CS.$$

Proof. Consider the delay differential system (2.1). Using the Lyapunov–Krasovskii functional, candidates are

$$V(t, x(t)) = v_1(t, x(t)) + v_2(t, x(t)) + v_3(t, x(t)) + v_4(t, x(t)), \tag{3.1}$$

where

$$v_1(t, x(t)) = e^{2\alpha t} x^T(t) P x(t), \tag{3.2}$$

$$v_2(t, x(t)) = \int_{-h(t)}^{0} e^{2\alpha(t+s)} x^T(t+s) Q x(t+s) ds, \tag{3.3}$$

$$v_3(t, x(t)) = \int_{-h(t)}^{0} e^{2\alpha(t+s)} \dot{x}^T(t+s) R \dot{x}(t+s) ds, \tag{3.4}$$

$$v_4(t, x(t)) = -h \int_{-h}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) S \dot{x}(s) ds d\theta. \tag{3.5}$$

Then the time derivative of $V(t, x(t))$ with respect to t along with the system is

$$\dot{V}(t, x(t)) = \dot{v}_1(t, x(t)) + \dot{v}_2(t, x(t)) + \dot{v}_3(t, x(t)) + \dot{v}_4(t, x(t)), \tag{3.6}$$

$$\begin{aligned}
\dot{v}_1(t, x(t)) =& 2\alpha e^{2\alpha t} x^T(t) P x(t) + e^{2\alpha t} x^T(t) P \dot{x}(t) + e^{2\alpha t} \dot{x}^T(t) P x(t) \\
=& e^{2\alpha t} x^T(t)(A^T P + PA + 2\alpha P) x(t) + x^T(t-h(t))(B^T P) x(t-h(t)) \\
& + \dot{x}^T(t-h(t))(C^T P) x(t) + x^T(t)(PB) x^T(t-h(t)) \\
& + x^T(PC) \dot{x}^T(t-h(t)), \tag{3.7}
\end{aligned}$$

$$\dot{v}_2(t, x(t)) = e^{2\alpha t}(x^T(t) Q x(t) - (1-d) e^{-2\alpha h} x^T(t-h) Q x(t-h(t))), \tag{3.8}$$

$$\begin{aligned}
\dot{v}_3(t, x(t)) =& e^{2\alpha t}(\dot{x}^T(t) R \dot{x}(t) - (1-d) e^{-2\alpha h} \dot{x}^T(t-h(t)) R \dot{x}(t-h(t))) \\
=& e^{2\alpha t} \Big(x^T(t) A^T A R x(t) + x^T(t) A^T B R x(t-h(t)) \\
& + x^T(t) A^T C R \dot{x}(t-h(t)) + x^T(t-h(t)) B^T B R x(t-h(t)) \\
& + x^T(t-h(t)) B^T A R x(t) + x^T(t-h(t)) B^T C R \dot{x}(t-h(t)) \\
& + \dot{x}^T(t-h(t)) C^T A R x(t) + \dot{x}^T(t-h(t)) C^T B R x(t-h(t)) \\
& + \dot{x}^T(t-h(t)) C^T C R \dot{x}(t-h(t)) \\
& - (1-d) e^{-2\alpha h} \dot{x}^T(t-h(t)) R \dot{x}(t-h(t)) \Big), \tag{3.9}
\end{aligned}$$

$$\dot{v}_4(t, x(t)) = e^{2\alpha t} \left(\dot{x}^T(t) h^2 S \dot{x}(t) - h \int_{t-h}^{t} e^{2\alpha(s-t)} \dot{x}^T(s) S \dot{x}(s) ds \right) \tag{3.10}$$

$$\begin{aligned}
= e^{2\alpha t} \Big(& \big(Ax(t) + Bx(t-h(t)) + C\dot{x}(t-h(t))\big)^T \\
& \times h^2 S(Ax(t) + Bx(t-h(t)) + C\dot{x}(t-h)) \\
& - h \int_{t-h}^{t} e^{2\alpha(s-t)} \dot{x}^T(s) S \dot{x}(s) ds \Big). \tag{3.11}
\end{aligned}$$

Obviously $s \in (t-h, t)$ and we have $e^{-2\alpha h} \le e^{2\alpha(s-t)}$ and

$$-h \int_{t-h}^{t} e^{2\alpha(s-t)} \dot{x}^T(s) S \dot{x}(s) ds \le -h e^{2\alpha h} \int_{t-h}^{t} \dot{x}^T(s) S \dot{x}(s) ds. \tag{3.12}$$

Using Lemma 2.2

$$-he^{2\alpha h}\int_{t-h}^{t}\dot{x}^T(s)S\dot{x}(s)ds \leq -e^{2\alpha h}(\int_{t-h}^{t}\dot{x}^T(s)ds)^TS(\int_{t-h}^{t}\dot{x}^T(s)ds) \qquad (3.13)$$
$$\leq -e^{2\alpha h}(x(t)-x(t-h(t)))^TS(x(t)-x(t-h(t))).$$

So

$$\begin{aligned}
\dot{v}_4(t,x(t)) =&e^{2\alpha t}\Big(x^T(t)(h^2A^TAS-e^{-2\alpha h}S)x(t)\\
&+x^T(t)(h^2A^TBS+e^{-2\alpha h}S)x(t-h(t))\\
&+x^T(t)(h^2A^TCS)\dot{x}(t-h(t))\\
&+x^T(t-h(t))(h^2B^TAS+e^{-2\alpha h}S)x(t)\\
&+x^T(t-h(t))(h^2B^TBS-e^{-2\alpha h}S)x(t-h(t))\\
&+x(t-h(t))(h^2SB^TC)\dot{x}(t-h(t))\\
&+\dot{x}^T(t-h(t))(h^2C^TAS)x(t)\\
&+\dot{x}^T(t-h(t))(h^2C^TBS)x(t-h(t))\\
&+\dot{x}^T(t-h(t))(h^2C^TCS)\dot{x}(t-h(t))\Big), \qquad (3.14)
\end{aligned}$$

then the complete derivative

$$\dot{V}(t,x(t)) = \dot{v}_1(t,x(t))+\dot{v}_2(t,x(t))+\dot{v}_3(t,x(t))+\dot{v}_4(t,x(t)) \qquad (3.15)$$

becomes

$$\begin{aligned}
\dot{V}(t,x(t)) =&e^{2\alpha t}\Big(x^T(t)(A^TP+PA+2\alpha P+Q+A^TAR+h^2A^TAS-e^{-2\alpha h}S)x(t)\\
&+x(t)(PB+A^TRB+h^2A^TBS+e^{-2\alpha h}S)x(t-h(t))\\
&+x^T(t)(PC+A^TRC+h^2A^TCS)\dot{x}(t-h(t))\\
&+x^T(t-h(t))(B^TP+B^TRA+h^2B^TAS+e^{-2\alpha h}S)x(t)\\
&+x^T(t-h(t))(B^TBR-(1-d)e^{-2\alpha h}Q\\
&\qquad\qquad +h^2B^TBS-e^{-2\alpha h}S)x(t-h(t))\\
&+x^T(t-h(t))(B^TCR+h^2SB^TC)\dot{x}(t-h(t))\\
&+\dot{x}^T(t-h(t))(C^TP+C^TAR+h^2C^TAS)x(t)\\
&+\dot{x}^T(t-h(t))(C^TBR+h^2C^TBS)x(t-h(t))\\
&+\dot{x}^T(t-h(t))(C^TCR-(1-d)e^{-2\alpha h}R+h^2C^TCS)\dot{x}(t-h(t))\Big).\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (3.16)
\end{aligned}$$

Furthermore, using the Schur-complement lemma, rewriting in terms of LMI's

of the form $\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{12}^T & \phi_{22} & \phi_{23} \\ \phi_{13}^T & \phi_{23}^T & \phi_{33} \end{bmatrix} < 0$, where

$$\phi_{11} = A^T P + PA + 2\alpha P + Q + A^T AR + h^2 A^T AS - e^{-2\alpha h} S,$$

$$\phi_{12} = PB + A^T RB + h^2 A^T BS + e^{-2\alpha h} S,$$

$$\phi_{13} = PC + A^T RC + h^2 A^T CS,$$

$$\phi_{22} = B^T BR - (1-d)e^{-2\alpha h} Q + h^2 B^T BS - e^{-2\alpha h} S,$$

$$\phi_{23} = B^T CR + h^2 SB^T C,$$

$$\phi_{33} = C^T CR - (1-d)e^{-2\alpha h} R + h^2 C^T CS.$$

Applying Lemma 2.3 in Φ with some effort, we get $\Phi < 0$. Therefore, by the Lyapunov–Krasovskii stability theorem $\dot{v}(t, x(t)) < 0$, we conclude the following result

$$\lambda_m(P)e^{2\alpha t}|x(t)|^2 \leq V(t) \leq V(0), \tag{3.17}$$

where

$$
\begin{aligned}
V(0) =& x^T(0)Px(0) + \int_{-h(0)}^0 e^{2\alpha s} x^T(s)Qx(s)ds \\
&+ \int_{-h(0)}^0 e^{2\alpha s}\dot{x}^T(s)R\dot{x}(s)ds + h\int_{-h}^0\int_{\theta)}^0 e^{2\alpha s}\dot{x}^T(s)S\dot{x}(s)ds = \lambda|(\mu)|_h^2
\end{aligned}
\tag{3.18}
$$

and

$$\lambda = \lambda_M(P) + h\lambda_M(Q) + h\lambda_M(R) + h^3\lambda_M(S).$$

Hence

$$\|x(t)\| \leq \sqrt{(\lambda_M(P) + h\lambda_M(Q) + h\lambda_M(R) + h^3\lambda_M(S))/\lambda_m(P)}|x(t)|_h e^{-\alpha t} \geq 0. \tag{3.19}$$

This implies that the system (2.1) is globally exponentially stable with convergence rate α. □

Theorem 3.2. *The system* (2.1) *is globally exponentially stable, if there exist positive-definite symmetric matrices* $P, Q, R, S \in \mathbb{R}^{n*n}$ *and positive scalar* α *satisfying the following generalized eigenvalue problem (GEVP):*

$$\text{Minimize } \alpha$$

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{12}^T & \phi_{22} & \phi_{23} \\ \phi_{13}^T & \phi_{23}^T & \phi_{33} \end{bmatrix} < 0,$$

$$\frac{1}{h}(P) < \alpha(K_1 + \frac{1}{h}(P)),$$

$$K_2 < \alpha Q,$$

$$K_3 < \alpha R,$$

$$K_4 < \alpha S,$$

where

$$\phi_{11} = A^T P + PA + K_1 + Q + A^T AR + h^2 A^T AS - K_4,$$

$$\phi_{12} = PB + A^T RB + h^2 A^T BS + K_4,$$

$$\phi_{13} = PC + A^T RC + h^2 A^T CS,$$

$$\phi_{22} = B^T BR - (1-d)K_2 + h^2 B^T BS - K_4,$$

$$\phi_{23} = B^T CR + h^2 SB^T C,$$

$$\phi_{33} = C^T CR - (1-d)K_3 + h^2 C^T CS.$$

Proof. The Lyapunov functions are chosen to be same as in Theorem 3.1 with

$$0 < 2\alpha P = (1 + 2\alpha P)\frac{1}{h}P - \frac{1}{h}P < e^{-2\alpha h} - \frac{1}{h}P = \frac{1}{\alpha}\frac{1}{h}P - \frac{1}{h}P < K,$$

$$0 < K_2 < \alpha Q = e^{-2\alpha h}Q,$$

$$0 < K_3 < \alpha R = e^{-2\alpha h}R,$$

$$0 < K_4 < \alpha S = e^{-2\alpha h}S.$$

The rest of the proof follows the same method as that in Theorem 3.1.   □

Remark 3.3. In this article we used the Lyapunov functions similar to [12] in order to prove exponential stability with convergence rate α but, using the Leibniz–Newton formula (Lemma 2.2) approach, we show the global exponential stability of the neutral delay differential system.

Remark 3.4. In order to solve the LMI in Theorem 3.1, we can utilize MATLAB LMI control toolbox [3].

Remark 3.5. In many existing papers, the assumption $\bar{h} \leq d < 1$ is needed; (see [5]), but in this paper this constraint is not necessary, which means that a fast time-varying delay is allowed.

4. Numerical examples

In this section, to illustrate the main result, we present the following simulation example.

Example 4.1. Consider the neutral delay-differential system (2.1) with the values from [12]

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

For $d = 0$ and $\alpha = 0.1$, applying Theorem 3.1, the following feasible solution is obtained by using MATLAB LMI toolbox. $P = 10^3 * \begin{bmatrix} 2.8690 & 0.0144 \\ 0.0144 & 2.5347 \end{bmatrix}$,

$$Q = 10^3 * \begin{bmatrix} 3.6151 & -0.0094 \\ -0.0094 & 3.6851 \end{bmatrix},$$

$$R = 10^3 * \begin{bmatrix} 1.2685 & -0.0039 \\ -0.0039 & 1.0790 \end{bmatrix}, S = \begin{bmatrix} 32.8051 & 0.2856 \\ 0.2856 & 28.0384 \end{bmatrix}.$$

TABLE 1.   Maximum allowable bound h for $\alpha = 0.1$ in Example 4.1

Methods	maximum upper bounded value h
[12]	0.7516
Theorem 3.1	1.6460e+003

Therefore, the concerned delay differential system with time-varying delays is globally exponentially stable.

Example 4.2. Consider the nominal time delay system from [11]

$$\dot{x}(t) = Ax(t) + Bx(t - h) \qquad (4.1)$$

with

$$A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} -0.5 & 0.1 \\ 0.3 & 0 \end{bmatrix}, C = 0.$$

For $d = 0$ and $\alpha = 0.5$, Theorem 3.1 the following feasible solution is obtained by using MATLAB LMI toolbox:

$$P = \begin{bmatrix} 186.6599 & 211.4091 \\ 211.4091 & 245.2739 \end{bmatrix}, \quad Q = 10^4 * \begin{bmatrix} 4.7559 & 3.7901 \\ 3.7901 & 5.6990 \end{bmatrix},$$

$$R = \begin{bmatrix} 9.6958 & 7.4502 \\ 7.4502 & 41.0085 \end{bmatrix}, \text{ and } \quad S = \begin{bmatrix} 0.3221 & 0.1310 \\ 0.1310 & 0.8085 \end{bmatrix}.$$

TABLE 2.   Maximum allowable bound h for $\alpha = 0.5$ in Example 4.2

Methods	maximum upper bounded value h
[2]	0.7062
[11]	1.8047
Theorem 3.1	8.0091e+003

Comparatively we got more conservative result than earlier, and the concerned delay differential system with time-delay is globally exponentially stable.

5. Conclusion

In this paper, we study about the sufficient conditions of exponential stability analysis for delay-differential systems of neutral type. The derived conditions are expressed in terms of LMI, using the Newton–Leibniz formula and Schur-complement lemma, two numerical examples with comparison with the existing results are given to demonstrate of our result with help of MATLAB LMI tool box.

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[1] Department of Mathematics, Dayananda sagar college of Engineering, Kumaraswamy layout, Bangalore, Karnataka, India.
E-mail address: vumeshakumar@gmail.com

[2] Department of Mathematics, R N S I T, Uttarahalli-Kengeri Road, Bangalore, Karnataka, India.
E-mail address: padmanabhanrnsit@gmail.com

[3] Department of Mathematics, New Horizon college of Engineering, Bangalore, Karnataka, India.
E-mail address: pbaskar83@yahoo.com

[4] Department of Pure Mathematics, Thiruvalluvar University, Vellore 632-115, Tamilnadu, India.
E-mail address: syedgru@gmail.com