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DYNAMIC RESPONSE OF AN ELASTICALLY CONNECTED DOUBLE NON-MINDLIN PLATES WITH SIMPLY-SUPPORTED END CONDITION DUE TO MOVING LOAD

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Abstract. In this paper, the dynamic response of two identical parallel non-mindlin (i.e., not taking into account the effect of shear deformation and rotatory inertia) plates which are elastically connected and subjected to a constant moving load is considered. The fourth order coupled partial differential governing equations is formulated and solved, using an approximate analytical method by assuming; firstly, a series solution later on treating the resulting coupled second order ordinary differential equations with an asymptotic method of Struble. The differential transform method, being a semi-analytical technique, is applied to the reduced coupled second order ordinary differential equations, to get a non-oscillatory series solution. An after treatment technique, comprising of the Laplace transform and Pade approximation techniques, is finally used via MAPLE ODE solver to make the series solution oscillatory. The dynamic deflections of the upper and lower plates are presented in analytical closed forms. The effect of the moving speed of the load and the elasticity of the elastic layer on the dynamic responses of the double plate systems is graphically shown and studied in details. The graphs of the plate's deflections for different speed parameters were plotted. It is however observed that the transverse deflections of each of the plates increase with an increase in different values of velocities for the moving load for a fixed time t.

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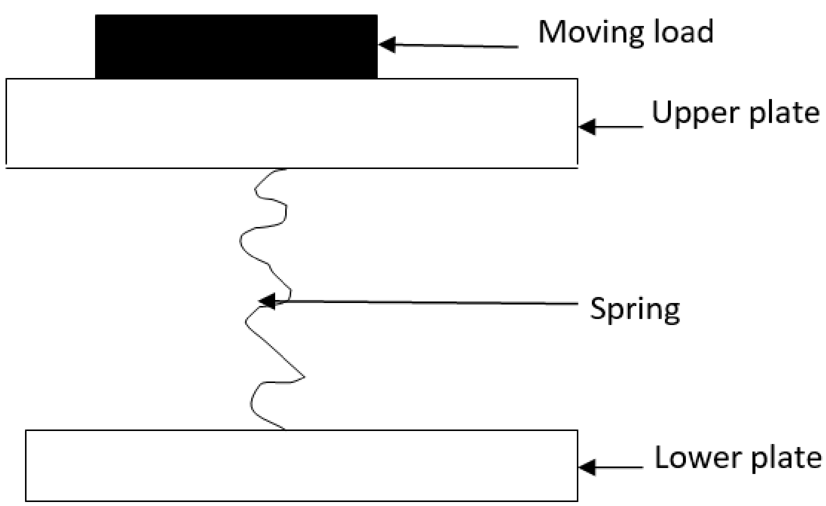
1. Introduction and preliminaries

Plates are solid structural elements having the geometry of two dimensions, whose thickness is very small compared to its planar dimensions. Also, the effects of load on the plates generate stresses normal to the thickness of the plate. In practical terms, the vibration analysis of plates have been explored and still being studied by many researchers in the field of applied mathematics and engineering, where, different constitutive governing equations with different end conditions were formulated. In several literature previously written, single beam or double beam systems, [4, 8, 12, 17, 21, 24] under moving loads acted upon by moving forces have been studied. But much work have not been carried out on double or multiplate systems, due to the problems encountered while trying to solve the coupled fourth order partial differential equations governing the systems. Fryba, [9] in his excellent monograph discussed the vibration of solids and structures under moving loads. Furthermore, Gbadeyan and Oni [12] considered the dynamic behavior of beams and rectangular plates under moving loads where it was shown that the response amplitude of the moving force is greater than that of the moving mass. Abu-Hilal [2], investigated the response of a double Euler–Bernoulli beams, due to a moving loads. Moreover, literatures of [5, 8, 10, 14, 15, 16, 19, 20, 22, 24] are work done on the theory and analysis of vibrations that are still producing positive development in the field of engineering. However, double-plate systems are very much applicable in engineering which includes, decking systems for railways viaducts and bridges, tunnels, anti slides and avalanche guards, industrial flooring systems, commercial flooring with high loading capacities, construction of trolleys and girders, and so many other applications. Also, its advantages include, provision of convenient transport, heavy loading capacity, great stability and long fatigue life, and so on. Having gone through the work of Gbadeyan and Hammed [11], where the influence of a moving mass of the dynamic behavior of a visco-elastically connected prismatic double Raighley beam system having arbitrary end support, in addition with the work of [3, 12, 13, 17, 18, 23], this paper considered, the problem of the effect of the influence of the mass of the moving load of constant magnitude and velocity of the dynamic behavior of the finite double elastically connected non-mindlin's plate with simply supported end conditions. The considered system is governed by a pair of fourth order partial differential equations, which is reduced to a pair of second order ordinary differential equations by using a pair of assumed series solutions. The reduced second order differential equation is further simplified, employing the approximate analytical method of Struble, which is commonly used to solve a weakly non-linear oscillatory system. The differential transform method (DTM) [1], is applied to obtain the solutions of the reduced coupled ordinary differential equations. Hence, a technique referred to as an after treatment (AT) [6, 7], comprising of the Laplace transform and Pade approximation is applied to the differential transform series solution, enlarging the convergence domain of the series solution by truncating it, thereby making the solution oscillatory. The simply supported end condition is considered as an example. The rest of the paper is organized into different sections which includes. Section 2 involves the mathematical model formulation with the corresponding boundary and initial conditions and defining the parameters involved in the systems. Section 3 involves the discussion of the method of solution . Section 4 presents the simplification of the coupled second order differential equations. Section 5 involves the solution of the two elastically connected double non-mindlin plate using DTM. Section 6, the simply supported condition is considered as an example. In section 7, AT technique is employed to obtain the solutions of upper plate $W_1(x, y; t)$ and lower plate $W_2(x, y; t)$ generated with computational software MAPLE ODE solver and numerical computations, is performed, and graphs were presented analytically and conclusively.

2. Mathematical model formulation

A double plate system modeled as a two finite parallel upper and lower non-mindlin plates inter-connected by an elastic core is considered. Figure 1, shows that the upper plate is subjected to a load $P_1(x, y; t)$ having a mass M_l

Figure 1. Diagram of the interconnected plate system



$$D\left[\frac{\partial^4 W_1(x,y;t)}{\partial x^4} + 2\frac{\partial^4 W_1(x,y;t)}{\partial x^2 \partial y^2} + \frac{\partial^4 W_1(x,y;t)}{\partial y^4}\right] + \mu\left[\frac{\partial^2 W_1(x,y;t)}{\partial t^2}\right] + K_1[\dot{W}_2(x,y;t) - \dot{W}_1(x,y;t)] = P_1(x,y;t) \tag{2.1}$$

$$D\left[\frac{\partial^4 W_1(x,y;t)}{\partial x^4} + 2\frac{\partial^4 W_1(x,y;t)}{\partial x^2 \partial y^2} + \frac{\partial^4 W_1(x,y;t)}{\partial y^4}\right] + \mu\left[\frac{\partial^2 W_1(x,y;t)}{\partial t^2}\right] + K_1[\dot{W}_2(x,y;t) - \dot{W}_1(x,y;t)] = 0. \tag{2.2}$$

The simply supported boundary conditions subjected to the pairs of fourth order coupled partial differential equations in (2.1) and (2.2) are:

$$\begin{aligned}
&W_1(l_x, y; t) = W_2(x, l_y; t) = 0\\
&W_1(0, y; t) = W_2(0, y; t) = 0\\
&\frac{\partial^2 W_1(0, y; t)}{\partial x^2} = \frac{\partial^2 W_1(x, y; t)}{\partial x^2} = 0\\
&\frac{\partial^2 W_1(0, y; t)}{\partial y^2} = \frac{\partial^2 W_1(x, y; t)}{\partial x^2} = 0.
\end{aligned} \tag{2.3}$$

While the corresponding initial conditions are:

$$\begin{cases} W_1(x, y; t)|_{t=0} = 0 = \dot{W}_1(x, y; t)|_{t=0}\\ W_2(x, y; t)|_{t=0} = 0 = \dot{W}_2(x, y; t)|_{t=0}. \end{cases} \tag{2.4}$$

Several parameters involved in the governing equations presented in (2.1)–(2.2) are defined as follows:

The concentrated moving load $P_1(x, y; t)$ is

$$\begin{aligned}
P_1(x, y; t) = \Bigg\{ M_l g - M_l \Bigg[&\frac{\partial^2 W_1(x, y; t)}{\partial t^2}\\
&+ 2V_1 \frac{\partial^2 W_2(x, y; t)}{\partial t \partial y} + V_1^2 \frac{\partial^2 W_1(x, y; t)}{\partial t^2} \Bigg] \Bigg\} \delta(x - vt)(y - s)
\end{aligned} \tag{2.5}$$

The Dirac delta function, being an even function expressed as a Fourier cosine series, is defined as:

$$\delta(x - vt) = \frac{1}{l_x} + \frac{2}{l_x} \sum_{n=1}^{\infty} \cos \frac{n\pi vt}{l_x} \cos \frac{n\pi x}{l_x} \text{ and } \delta(y - s) = \frac{1}{l_y} + \frac{2}{l_y} \sum_{m=1}^{\infty} \cos \frac{m\pi s}{l_y} \cos \frac{m\pi y}{l_y},$$

where

$$\begin{aligned}
\delta(x - vt)\delta(y - s) = &\frac{4}{l_x l_y} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \cos \frac{n\pi vt}{l_x} \cos \frac{m\pi s}{l_y} \cos \frac{n\pi vt}{l_x} \cos \frac{m\pi y}{l_y}\\
&+ \frac{2}{l_x l_y} \sum_{m=1}^{\infty} \cos \frac{n\pi vt}{l_x} \cos \frac{n\pi x}{l_x} + \frac{2}{l_x l_y} \sum_{m=1}^{\infty} \cos \frac{m\pi s}{l_y} \cos \frac{m\pi y}{l_y} + \frac{1}{l_x l_y},
\end{aligned} \tag{2.6}$$

where

$$\delta(x) = \begin{cases} 0, & x \neq 0,\\ \infty, & x = 0. \end{cases}$$

D is the flexural rigidity of the plate given as $D = \frac{Eh^3}{12(1-\theta)}$, and M_l is the mass of the load $P_1(x, y; t)$ moving with a constant velocity V, also E is Young's modulus, θ is Poisson's ratio ($\theta < 1$), x and y are the position coordinate in x and y direction, K is the stiffness constant, μ is the constant mass per unit length of the plates, ϵ is the fixed length of the plates, and t is the time. $W_1(x, y; t)$ is the traversed displacement of the upper plate and $W_2(x, y; t)$ is also the traversed displacement of the lower plate. Also, from (2.5), g is the acceleration due to gravity, $V_1^2\ W_1''(x,y;t)$ represents the centrifugal acceleration, $2VW_1'(x,y;t)$ is the Coriolis acceleration, while $\ddot{W}_1(x,y;t)$ represents the local acceleration.

3. Method of solution

To solve the transverse dynamic responses $W_1(x,y;t)$ and $W_2(x,y;t)$, (2.1)–(2.2) are transformed into a set of two ordinary differential equations. The method of solution of employed in this paper, involves assuming the series of the form:

$$\begin{aligned} W_1(x,y;t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\phi_{m,n}\ (m,n;t)V_m(x)V_n(y), \\ W_2(x,y;t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\beta_{m,n}\ (m,n;t)V_m(x)V_n(y), \end{aligned} \tag{3.1}$$

where

$$\begin{aligned}
\frac{\partial^4 W_1(x,y;t)}{\partial x^4} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{m,n}(m,n;t)V_m^{iv}(x)V_n(y), \\
\frac{\partial^4 W_1(x,y;t)}{\partial y^4} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{m,n}(m,n;t)V_m(x)V_n^{iv}(y), \\
\frac{\partial^4 W_1(x,y;t)}{\partial x^2\partial y^2} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{m,n}(m,n;t)V_m''(x)V_n''(y), \\
\frac{\partial^2 W_2(x,y;t)}{\partial t^2} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\ddot{\beta}_{m,n}(m,n;t)V_m(x)V_n(y), \\
\frac{\partial^2 W_1(x,y;t)}{\partial t^2} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\ddot{\phi}_{m,n}(m,n;t)V_m(x)V_n(y), \\
\frac{\partial^2 W_2(x,y;t)}{\partial x^2} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\ddot{\beta}_{m,n}(m,n;t)V_m''(x)V_n''(y), \\
\frac{\partial^4 W_2(x,y;t)}{\partial x^2\partial y^2} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\beta_{m,n}(m,n;t)V_m''(x)V_n''(y), \\
\frac{\partial^4 W_1(x,y;t)}{\partial y^4} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\beta_{m,n}(m,n;t)V_m(x)V_n^{iv}(y), \\
\frac{\partial^4 W_1(x,y;t)}{\partial t\partial x} &= \sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\dot{\phi}_{k,l}(k,l;t)V_k'(x)V_l(y), \\
\frac{\partial^2 W_1(x,y;t)}{\partial x^2} &= \sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\phi_{k,l}(k,l;t)V_k''(x)V_l''(y).
\end{aligned} \tag{3.2}$$

Substituting (3.2) into (2.1) yields

$$\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{m,n}(m,n;t)V_k^{iv}(x)V_n(y)+2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{m,n}(m,n;t)V_m''(x)V_n''(y)\right.$$
$$\left.+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{m,n}(m,n;t)V_m(x)V_n^{iv}(y)+\mu\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{m,n}(m,n;t)V_m''(x)V_n''(y)\right]$$
$$+K_1\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\phi_{m,n}(m,n;t)V_m(x)V_n(y)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\beta_{m,n}(m,n;t)V_m(x)V_n(y)\right]$$
$$=M_lg-M_l\left[\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\ddot{\phi}_{k,l}(k,l;t)V_k'(x)V_l(y)+2V_1\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\dot{\phi}_{k,l}(k,l;t)V_k'(x)V_l(y)\right.$$
$$\left.+V_1^2\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\phi_{k,l}(k,l;t)V_k''(x)V_l''(y)\right]\delta(x-vt)(y-s). \tag{3.3}$$

We assume that the load function $P_1(x,y;t)$ can also be expressed as

$$P_1(x,y;t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\phi_{m,n}(m,n;t)V_m(x)V_n(y), \tag{3.4}$$

where, $\phi_{m,n}(m,n;t)$, $\beta_{m,n}(m,n;t)$, and $\psi_{m,n}(m,n;t)$ are unknown functions of time t and $V_m(x)$, $V_n(y)$ are the known deflection curves of a vibrating plate. For the free vibration of a rectangular plate, we have, for the homogeneous part of the governing equation in terms of the assumed solution,

$$D\left[V_m^{iv}(x)V_n(y)+2V_m''(x)V_n''(y)+V_m(x)V_n^{iv}(y)\right]-\mu\omega_{m,n}^2V_m(x)V_n(y)=0, \tag{3.5}$$

where the frequency $\omega_{m,n}^2$ for a simply supported plate is

$$\omega_{m,n}^2=\left[\frac{m^4\pi^4}{L_x^4}+2\frac{m^2n^2\pi^2}{L_x^2L_y^2}+\frac{n^4\pi^4}{L_y^4}\right]. \tag{3.6}$$

Also, the Kth mode of vibration of a uniform plate in x-direction is given as

$$V_k(x)=\sin\frac{\lambda_k}{l_x}(x)+A_k\cos\frac{\lambda_k}{l_x}(x)+B_k\sinh\frac{\lambda_k}{l_x}(x)+C_k\cosh\frac{\lambda_k}{l_x}(x).$$

While the corresponding Lth mode of vibration of a uniform plate in x and also y direction are

$$V_l(x)=\sin\frac{\lambda_y}{l_y}(y)+A_k\cos\frac{\lambda_y}{l_y}(y)+B_k\sinh\frac{\lambda_y}{l_y}(y)+C_k\cosh\frac{\lambda_y}{l_y}(y)$$

$$V_l(y)=\sin\frac{\lambda_y}{l_y}(y)+A_k\cos\frac{\lambda_y}{l_y}(y)+B_k\sinh\frac{\lambda_y}{l_y}(y)+C_k\cosh\frac{\lambda_y}{l_y}(y), \tag{3.7}$$

where the constants A_k, B_k, C_k, and λ_k in (3.7) are determined by the simply supported boundary conditions.

After rearrangement, simplification and modification, (2.1) reduces to

$$\begin{aligned}&\ddot{\phi}_{m,n}(m,n;t)+\omega^2_{m,n}(m,n;t)\phi_{m,n}(m,n;t)+\frac{K_1}{\mu}\phi_{m,n}(m,n;t)\\&+\frac{K_1}{\mu}\beta_{m,n}(m,n;t)+\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}[q_l^*(a,b;t)+Vq_2^*(a,b;t)+Vq_3^*(a,b;t)]\\&\qquad=\frac{M_gV_mvtV_mS}{\mu}.\end{aligned}\tag{3.8}$$

Following the same procedure for the lower plate $W_2(x,y;t)$, (2.2) becomes

$$\ddot{\beta}_{m,n}(m,n;t)+\omega^2_{m,n}(m,n;t)\beta_{m,n}(m,n;t)+\frac{K_1}{\mu}\phi_{m,n}(m,n;t)-\frac{K_1}{\mu}\beta_{m,n}(m,n;t)=0,\tag{3.9}$$

where

$$\begin{aligned}q_l^*(a,b;t)=&4\epsilon\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\ddot{\phi}_{k,l}(m,n;t)\cos\frac{n\pi vt}{l_x}\cos\frac{m\pi s}{l_y}\Delta_q(R,S)\Delta_t(U,V)\\&+2\epsilon\sum_{k=1}^{\infty}\ddot{\phi}_{k,l}(m,n;t)\cos\frac{n\pi vt}{l_x}\Delta_q(R,S)\nabla_1(U,V)\\&+2\epsilon\sum_{m=1}^{\infty}\ddot{\phi}_{k,l}(m,n;t)\cos\frac{m\pi s}{l_x}\Delta_t(U,V)\nabla_1(J,K)\\&+\cos\frac{m\pi s}{l_x}\Delta_t(U,V)\nabla_1(J,K)+\epsilon\ddot{\phi}_{k,l}\Delta_1(J,K)\nabla_1(U,V),\end{aligned}\tag{3.10}$$

$$\begin{aligned}q_2^*(a,b;t)=&8\epsilon\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\phi_{k,l}(m,n;t)\cos\frac{n\pi vt}{l_x}\cos\frac{m\pi s}{l_y}\Delta_{2c}(J,K)\Delta_t(U,V)\\&+4\epsilon\sum_{k=1}^{\infty}\phi_{k,l}(m,n;t)\cos\frac{n\pi vt}{l_x}\Delta_{2c}(J,K)\nabla_t(U,V)\\&+4\epsilon\sum_{m=1}^{\infty}\phi_{k,l}(m,n;t)\cos\frac{m\pi s}{l_y}\Delta_t(U,V)\nabla_2(J,K)\\&+\cos\frac{m\pi s}{l_x}\Delta_t(U,V)\nabla_1(J,K)+\epsilon\phi_{k,l}\Delta_1(J,K)\nabla_1(U,V),\end{aligned}\tag{3.11}$$

$$\begin{aligned}q_3^*(a,b;t)=&4\epsilon\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\phi_{k,l}(m,n;t)\cos\frac{n\pi vt}{l_x}\cos\frac{m\pi s}{l_y}\Delta_{3c}(J,K)\Delta_k(U,V)\\&+2\epsilon\sum_{k=1}^{\infty}\phi_{k,l}(m,n;t)\cos\frac{n\pi vt}{l_x}\Delta_{3c}(J,K)\nabla_1(U,V)\\&+2\epsilon\sum_{m=1}^{\infty}\phi_{k,l}(m,n;t)\cos\frac{m\pi s}{l_y}\Delta_t(U,V)\nabla_3(J,K)+\epsilon\phi_{k,l}\Delta_1(J)\nabla_1(P).\end{aligned}\tag{3.12}$$

The following are defined as

$$
\begin{aligned}
\Delta_q(R,S) &= \int_0^{l_x} \cos\frac{n\pi x}{l_x} V_k(x)V_j(x)dx,\\
\Delta_t(U,V) &= \int_0^{l_y} \cos\frac{m\pi y}{l_x} V_l(y)V_p(y)dy,\\
\Delta_1(J,K) &= \int_0^{l_x} V_k(x)V_j(x)dx,\\
\nabla_1(U,V) &= \int_0^{l_y} V_l(x)V_p(x)dy,\\
\Delta_{2c}(J,K) &= \int_0^{l_x} \cos\frac{n\pi x}{l_x} V_k'(x)V_j(x)dx,\\
\Delta_2(J,K) &= \int_0^{l_x} V_k'(x)V_j(x)dx,\\
\Delta_{3c}(J,K) &= \int_0^{l_x} \cos\frac{n\pi x}{l_x} V_k''(x)V_j(x)dx,\\
\Delta_3(J,K) &= \int_0^{l_x} V_k''(x)V_j(x)dx,
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta_k(J) &= \int_0^{l_x} \cos\frac{n\pi x}{l_x} V_j(x)dx,\\
\nabla_k(P) &= \int_0^{l_y} \cos\frac{m\pi y}{l_x} V_p(y)dy,\\
\Delta_1(J) &= \int_0^{l_x} V_j(x)dx,\\
\nabla_1(P) &= \int_0^{l_y} V_p(y)dy.
\end{aligned} \tag{3.13}
$$

.

Equations (3.8)–(3.13) are the coupled transformed second order ordinary differential equations governing the behavior of a double non-mindlin plate system interconnected by an elastic core traversed by a moving load. To be able to simplify further, by (3.8) and (3.9), we assume, for the time being, that the plate is disconnected. That is, we set the connecting elastic core term to zero, to become;

$$\frac{K_1}{\mu}\left[\phi_{m,n}(m,n;t) - \beta_{m,n}(m,n;t)\right] = 0. \tag{3.14}$$

So that the resulting equations for the upper and lower plates are such that

$$\ddot{\phi}_{m,n}(m,n;t)+\omega^2_{m,n}(m,n;t)\phi_{m,n}(m,n;t) + \epsilon_1 \sum_{k=1}^{\infty}\sum_{l=1}^{\infty}[q^*_l(a,b;t)+Vq^*_2(a,b;t)+Vq^*_3(a,b;t)] = \frac{M_g V_m vtV_m S}{\mu} \tag{3.15}$$

and

$$\ddot{\beta}_{m,n}(m,n;t)+\omega^2_{m,n}(m,n;t)\beta_{m,n}(m,n;t)=0, \tag{3.16}$$

respectively.

4. Simplification of the coupled second order differential equations

We have been able to reduce the coupled fourth order partial differential equation to a second order differential equations, using an assumed series solution method. However, it is still observed that while the reduced lower plate equation can be solved analytically, the upper plate has no exact analytical solution, and to this effect, an approximate analytical method, which is the modified asymptotic technique developed by Struble was employed. We set

$$\lambda = \frac{\epsilon_1}{1+\epsilon_1} < 1, \tag{4.1}$$

and it can also be shown that

$$\epsilon_1 = \lambda + 0(\lambda^2) \tag{4.2}$$

and

$$\frac{1}{1+\epsilon_1 R_a t} = [1+\lambda R_a t]^{-1} = 1-\lambda R_a t + (\lambda)^2. \tag{4.3}$$

.

Substituting (4.3) into the homogeneous part of (3.15), we obtain

$$
\begin{aligned}
\ddot{\phi}_{m,n}(m,n;t) &+ \epsilon_1 R_a t(1-\lambda R_a t)\ddot{\phi}_{m,n}(m,n;t) \\
&+ (\omega_{m,n}^2 + \epsilon_1 R_c t)(1-\lambda R_a t)\phi_{m,n}(m,n;t) \\
&+ \lambda(1-\lambda R_a t)\sum_{k=1 k\neq m}^{\infty}\sum_{l=1 l\neq n}^{\infty} 4\cos\frac{n\pi vt}{l_x}\cos\frac{m\pi s}{l_y}\Delta_q(R,S)\nabla_1(U,V) \\
&+ 2\sum_{n=1}^{\infty}\cos\frac{n\pi vt}{l_x}\Delta_q(R,S)\nabla_1(U,V) \\
&+ 2\sum_{m=1}^{\infty}\cos\frac{n\pi vt}{l_x}\Delta_t(U,V)\nabla_1(J,K) \\
&+ \sum_{k=1}^{\infty}\Delta_1(J,K)\nabla_1(U,V)\ddot{\phi}_{k,l}(k,l;t) \\
&+ 8V\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\cos\frac{n\pi vt}{l_x}\cos\frac{m\pi s}{l_y}\Delta_{2c}(J,K)\nabla_t(U,V) \\
&+ 4V\sum_{k=1}^{\infty}\cos\frac{n\pi vt}{l_x}\Delta_{2c}(J,K)\nabla_t(U,V) \\
&+ 4V\sum_{k=1}^{\infty}\cos\frac{m\pi s}{l_y}\Delta_t(U,V)\nabla_2(J,K) \\
&+ 2V\Delta_2(J,K)\nabla_1(U,V)\dot{\phi}_{k,l}(k,l;t) \\
&+ 4V^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\cos\frac{n\pi vt}{l_x}\cos\frac{m\pi s}{l_y}\Delta_{3c}(J,K)\nabla_k(P) \\
&+ 2V^2\sum_{n=1}^{\infty}\cos\frac{n\pi vt}{l_x}\Delta_{3c}(J,K)\nabla_1(U,V) \\
&+ 2V^2\sum_{m=1}^{\infty}\cos\frac{n\pi vt}{l_y}\cos\frac{m\pi s}{l_y}\Delta_{3c}(U,V)\nabla_2(J,K) \\
&+ \nabla_1(P)\Delta_1(J)\dot{\phi}_{k,l}(k,l;t) = 0.
\end{aligned}
\tag{4.4}
$$

Setting $\lambda = 0$ in (4.4), we obtain a case corresponding to that when the mass ratio effect of the plates is regarded negligible. In this case we have

$$
\begin{aligned}
\ddot{\phi}_{m,n}(m,n;t) &+ \epsilon_1 R_a t\dot{\phi}_{m,n}(m,n;t) \\
&+ \omega_{m,n}^2(m,n;t)\beta_{m,n}(m,n;t) + [\epsilon_1 R_c t]\dot{\phi}_{m,n}(m,n;t) = 0,
\end{aligned}
\tag{4.5}
$$

whose solution is of the form

$$
\ddot{\phi}_{m,n}(m,n;t) = D_0[\omega_{m,n}t - \alpha_{m,n}], \tag{4.6}
$$

where D_0, $\omega_{(m,n)}$, and $\alpha_{(m,n)}$ in (4.6) are all constants. Since $\lambda = 1$, for any arbitrary mass ratio ϵ_1, Struble's method requires that the asymptotic solution of the (4.4) is of the form

$$\phi_{m,n}(t) = \theta(m,n;t)\cos[\omega_{m,n}t - \alpha_{m,n}] + \sum_{n=1}^{\overline{N}} \lambda^r \phi_r(m,n;t) + 0(\lambda^{\overline{N}}), \tag{4.7}$$

where N is any positive number less than infinity and $\theta(m,n;t)$ and $\alpha(m,n;t)$ are slowly varying time functions, which implies that,

$$\begin{aligned} \dot{\theta}(m,n;t) &\to 0(\lambda), \\ \ddot{\theta}(m,n;t) &\to 0(\lambda^2), \\ \dot{\alpha}(m,n;t) &\to 0(\lambda), \\ \ddot{\alpha}(m,n;t) &\to 0(\lambda^2). \end{aligned} \tag{4.8}$$

Substituting, without loss of generality, $\overline{N} = 1$, it becomes

$$\phi_{m,n}(t) = \theta(m,n;t)\cos[\omega_{m,n}t - \alpha_{m,n}] + \lambda\phi_1(m,n;t) + 0(\lambda^2). \tag{4.9}$$

Hence

$$\phi_{m,n}(t) = \theta(m,n;t)\cos[\omega_{m,n}t - \alpha_{m,n}] + \lambda\phi_1(m,n;t) + 0(\lambda^2) \tag{4.10}$$

such that

$$\begin{aligned} \dot{\phi}_{m,n}(t) =& \dot{\theta}(m,n;t)\cos[\omega_{m,n}t - \alpha_{m,n}] \\ &- \theta(m,n;t)\sin[\omega_{m,n}t - \alpha_{m,n}] + \lambda\dot{\phi}_1(m,n;t) \\ &+ 0(\lambda^2)\dot{\phi}_1(m,n;t) \end{aligned} \tag{4.11}$$

and

$$\begin{aligned} \ddot{\phi}_{m,n}(t) =& [\ddot{\theta}(m,n;t) - \omega_{m,n}^2(m,n;t) + 2\omega_{m,n}\theta(m,n;t)\dot{\alpha}(m,n;t) \\ &- \omega_{m,n}(t) - \dot{\alpha}_{m,n}^2(m,n;t)]\cos[\omega_{m,n}t - \alpha_{m,n}] - 2\omega_{m,n}\dot{\theta}(m,n;t)\dot{\alpha}(m,n;t) \\ &- 2\dot{\theta}(m,n;t)\ddot{\alpha}(m,n;t) - \theta(m,n;t)\ddot{\alpha}(m,n;t)]\sin[\omega_{m,n}t - \alpha_{m,n}] \\ &+ 0(\lambda^2)\dot{\phi}_1(m,n;t). \end{aligned} \tag{4.12}$$

The variational equations are obtained, by equating the coefficients of $\sin[\omega_{m,n}t - \alpha_{m,n}(m,n;t)]$ and $\cos[\omega_{m,n}t - \alpha_{m,n}(m,n;t)]$ terms on both sides of the (4.12) to zero. Neglecting those terms that do not contribute to the variational equations, we obtain

$$\begin{aligned} -2\omega_{m,n}&\dot{\theta}(m,n;t)\sin[\omega_{m,n}t - \alpha_{m,n}] \\ &+ 2\omega_{m,n}\theta(m,n;t)\dot{\alpha}(m,n;t)\cos[\omega_{m,n}t - \alpha_{m,n}] \\ &- 2\lambda V\omega_{m,n}\theta(m,n;t)\Delta_2(J,K)\nabla_1(U,V)\sin[\omega_{m,n}t - \alpha_{m,n}] \\ &- \lambda\omega_{m,n}^2\Delta_1(J,K)\nabla_1(U,V)\theta(m,n;t)\cos[\omega_{m,n}t - \alpha_{m,n}] \\ &- \lambda V^2\Delta_{2c}(J,K)\nabla_t(U,V)\theta(m,n;t)\cos[\omega_{m,n}t - \alpha_{m,n}(m,n;t)] = 0, \end{aligned} \tag{4.13}$$

where the variational equations are

$$-2\omega_{m,n}\dot{\theta} - 2\lambda V\omega_{m,n}\theta(m,n;t)\Delta_2(J,K)\nabla_1(U,V) = 0 \tag{4.14}$$

and

$$
\begin{aligned}
2\omega_{m,n}\theta\dot{\alpha}(m,n;t) - \lambda\alpha^2\omega_{m,n}\Delta_2(J,K)\nabla_1(U,V)\theta(m,n;t) \\
+ \lambda V^2\Delta_{2c}(J,K)\nabla_t(U,V)\theta(m,n;t) = 0
\end{aligned} \tag{4.15}
$$

solving (4.14) and (4.15), we obtain

$$
\dot{\theta}(m,n;t) = -\lambda\theta(m,n;t)\Delta_2(J,K)\nabla_1(U,V), \tag{4.16}
$$

That is,

$$
\frac{d\theta(m,n;t)}{\theta} = \lambda\Delta_2(J,K)\nabla_1(U,V)\theta, \tag{4.17}
$$

which implies that,

$$
\theta(m,n;t) = \epsilon^{-\lambda V\Delta_2(J,K)\nabla_1(U,V)} = A^0\epsilon^{-r^0 t}, \tag{4.18}
$$

where

$$
r^0 = \lambda V\Delta_2(J,K)\nabla_1(U,V) \tag{4.19}
$$

and A^0 is constant.
Equation (4.18) implies that,

$$
\begin{aligned}
&\frac{d\alpha(m,n;t)}{dt} \\
&= \frac{\lambda\omega_{m,n}\theta(m,n;t)}{2}\left[\Delta_1(J,K)\nabla_1(U,V) - \frac{\lambda}{V^2}2\alpha(m,n)\Delta_3(J,K)\nabla_1(U,V)\right]t + \phi_{m,n},
\end{aligned} \tag{4.20}
$$

where ϕ is constant. The first approximation to the homogeneous system is from (4.6)

$$
\phi_{m,n}(m,n;t) = D_0\cos[\omega_{m,n}t - \alpha_{m,n}(m,n;t)]. \tag{4.21}
$$

That is,

$$
\phi_{m,n}(m,n;t) = A^0\epsilon^{-r^0 t}\cos[\delta_{m,n}t - \psi_{m,n}(m,n;t)], \tag{4.22}
$$

where

$$
\delta_{m,n} = \left[1 - \frac{\lambda}{2}\Delta_1(J,K)\nabla_1(U,V) - \frac{V^2}{\omega_{m,n}^2}\Delta_3(J,K)\nabla_1(U,V)\right]. \tag{4.23}
$$

Equation (4.23) is the desired modified frequency corresponding to the frequency of free system involving moving mass effect. Hence, according to Struble's technique, (3.15) is reduced to

$$
\ddot{\phi}_{m,n}(m,n;t) + \delta_{m,n}^2\phi_{m,n}(m,n;t) = P_{RT}V_m(vt)V_n(s), \tag{4.24}
$$

where $P_{RT} = g\epsilon_1 l_x l_y(1 - \epsilon_1 R_a(t))$.

5. Solution of two elastically connected double non-mindlin plates

At this juncture, the connecting elastic core is restored and the reduced pair of coupled second order differential equation for the non-mindlin plates

$$
\begin{aligned}
\ddot{\phi}_{m,n}(m,n;t) + \omega_{m,n}^2\phi_{m,n}(m,n;t)\left[\frac{K_1}{\mu}\phi_{m,n}(m,n;t) - \frac{K_1}{\mu}\beta_{m,n}(m,n;t)\right] \\
= P_{RT}V_m(vt)V_n(s)
\end{aligned} \tag{5.1}
$$

and

$$\ddot{\phi}_{m,n}(m,n;t) + \delta^2_{m,n}\phi_{m,n}(m,n;t)\left[\frac{K_1}{\mu}\phi_{m,n}(m,n;t) - \frac{K_1}{\mu}\beta_{m,n}(m,n;t)\right] = 0 \quad (5.2)$$

in terms of the modified frequencies $\delta^2_{m,n}$ and $\omega^2_{m,n}$ are considered, respectively. A semi analytical method known as DTM is now employed. To this end, we will state briefly, the basic theory of method as follows. The transform of the mth derivative of a function $W(t)$ is given as

$$W(m) = \frac{1}{m!}\left[\frac{d^m W(t)}{dt^2}\right] \quad (5.3)$$

and the corresponding inverse transformation is defined as

$$W(t) = \sum_{m=1}^{\infty}(t - t_0)\, W(m). \quad (5.4)$$

Hence, the above equations will yield

$$W(t) = \sum_{m=1}^{\infty}\frac{t - t_0}{m!}\left[\frac{d^m W(t)}{dt^2}\right]. \quad (5.5)$$

Table 1. Basic properties of DTM for equations of motion

Original Function	**Transformed Function**
$w(t) = c_1u(t) \pm c_2v(t)$	$W(k) = c_1U(k) \pm c_2V(k)$
$w(t) = \frac{du(t)}{dt}$	$W(k) = (k+1)U(k+1)$
$w(t) = \frac{d^r u(t)}{dt^r}$	$W(k) = (k+1)(k+2)\cdots(k+n)U(k+n)$
$w(t) = u(t)v(t)$	$W(k) = \sum_{r=0}^{k} U(n)V(k-n)$
$w(t) = u(t)v(t)y(t)$	$W(k) = \sum_{n=0}^{k}\sum_{r=0}^{k-n} U(n)V(r)V(k-n-r)$
$w(t) = t^m$	$W(k) = \delta(k-m) = \begin{cases}1\\0\end{cases}$ $k = m$ and $k \neq m$
$w(t) = \sin at$	$W(k) = \frac{1}{k!}a^k \sin\left(\frac{k\pi}{2}\right)$
$w(t) = \cos at$	$W(k) = \frac{1}{k!}a^k \cos\left(\frac{k\pi}{2}\right)$
$w(t) = \sinh at$	$W(k) = \frac{1}{2k!}\left[a^k - (-(a)^k)\right]$
$w(t) = \cosh at$	$W(k) = \frac{1}{2k!}\left[a^k + (-(a)^k)\right]$

It is well known that in application the series in (5.4) is finite and usually written as

$$W(t) = \sum_{m=1}^{p} (t - t_0)^m W(m), \tag{5.6}$$

. such that the series

$$W(t) = \sum_{m=1}^{p+1} (t - t_0) \tag{5.7}$$

is considered unimportantly small. Furthermore, it can be readily shown that the relationships in Table 1 between the original function $W(t)$ and the transformed function $W(m)$, for $t_0 = 0$, hold. Applying the DTM on (5.1) and (5.2), gives

$$\begin{aligned}
\phi_{m,n}(k+2) = {} & \frac{1}{(k+1)(k+2)} P_{rt} V_n(s) \\
& \times \left[\frac{1}{k!} \left(\frac{\lambda_m V}{L} \right)^k \sin\left(\frac{k\pi}{2} \right) \frac{A_m}{k!} \left(\frac{\lambda_m V}{L} \right)^k \cos\left(\frac{k\pi}{2} \right) \right. \\
& \qquad \left. + \frac{B_m}{2k!} \left(\frac{\lambda_m V}{L} \right)^k \right] \\
& - \left[- \left(\frac{\lambda_m V}{L} \right)^k \right] + \frac{C_m}{2k!} \left(\frac{\lambda_m V}{L} \right)^k - \left[- \left(\frac{\lambda_m V}{L} \right)^k \right] - \delta^2_{m,n} \phi(k) \\
& - \frac{K_1}{\mu} \phi_{m,n}(m,n;t) + \frac{K_1}{\mu} \beta_{m,n}(m,n;t)
\end{aligned} \tag{5.8}$$

$$\begin{aligned}
\beta_{m,n}(k+2) = {} & \frac{1}{(k+1)(k+2)} \\
& \left\{ \omega^2_{m,n} - \beta_{m,n}(m,n;t) - \frac{K_1}{\mu} \beta_{m,n}(m,n;t) + \frac{K_1}{\mu} \phi_{m,n}(m,n;t) \right\}
\end{aligned} \tag{5.9}$$

Subjecting (5.8) and (5.9) to the transformed form of the initial conditions earlier stated in (2.4), our assumed solutions earlier stated in (3.2) will become

$$\begin{aligned}\overline{W}_1(x,y;t) =& \sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{P_{RT}V_n(s)}{\alpha_m^2-\delta_{m,n}^2}\frac{1}{2!}\left(A_m+C_m\right)\left(\alpha_m^2-\delta_{m,n}^2\right)t^2\\ &+\frac{1}{3!}\left(\alpha_m^2-\delta_{m,n}^2\right)\left(\alpha(1+B_m)\right)t^3\\ &+\frac{1}{4!}\left[\left(A_m+C_m\right)\left(\alpha_m^2-\delta_{m,n}^2\frac{K_1}{\mu}\right)\right]t^4\\ &+\frac{1}{5!}\left[\left(A_m+C_m\right)\left(\alpha_m^2-\delta_{m,n}^2\frac{K_1}{\mu}\right)\right](\alpha_m 1+B_mV_ns)t^5\\ &\times\left[\sin\frac{\lambda_m x}{l_x}+A_m\cos\frac{\lambda_m x}{l_x}+B_m\sinh\frac{\lambda_m x}{l_x}+C_m\cosh\frac{\lambda_m x}{l_x}\right]\\ &\quad\times\left[\sin\frac{\lambda_n y}{l_y}+A_n\cos\frac{\lambda_n x}{l_n}+B_n\sinh\frac{\lambda_n y}{l_y}+C_n\cosh\frac{\lambda_n y}{l_y}\right],\end{aligned}\tag{5.10}$$

.

$$\begin{aligned}\overline{W}_1(x,y;t) =& \sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{P_{RT}V_n(s)}{\alpha_m^2-\delta_{m,n}^2}\left[\frac{1}{4!}\frac{K_1}{\mu}\left(A_m+C_m\right)\left(\alpha_m^2-\delta_{m,n}^2\frac{K_1}{\mu}\right)\right]t^4\\ &+\frac{1}{5!}\frac{K_1}{\mu}\left[\alpha(1+B_m)\left(\alpha_m^2-\delta_{m,n}^2\right)\right](\alpha_m 1+B_mV_ns)t^5\\ &\quad\times\left[\sin\frac{\lambda_m x}{l_x}+A_m\cos\frac{\lambda_m x}{l_x}+B_m\sinh\frac{\lambda_m x}{l_x}+C_m\cosh\frac{\lambda_m x}{l_x}\right]\\ &\quad\times\left[\sin\frac{\lambda_n y}{l_y}+A_n\cos\frac{\lambda_n x}{l_n}+B_n\sinh\frac{\lambda_n y}{l_y}+C_n\cosh\frac{\lambda_n y}{l_y}\right].\end{aligned}\tag{5.11}$$

Therefore, (5.10) and (5.11) represent the transverse displacement of the non-mindlin's plates interconnected by an elastic layer and traversed by a moving mass and having arbitrary end supports.

6. Simply-supported boundary condition as illustrative example

The considered system is made up of two plates interconnected by an elastic core. The eigen functions or normal modes are:

$$\begin{aligned}V_m(0)&=0=V_m(l_x),\\ V_m''(0)&=0=V_m''(l_x),\end{aligned}\tag{6.1}$$

as well as,

$$\begin{aligned}V_n(0)&=0=V_n(l_x),\\ V_n''(0)&=0=V_n''(l_x).\end{aligned}\tag{6.2}$$

Using the boundary conditions earlier stated in (2.4), we obtain

$$\begin{aligned} A_m = B_m = C_m = A_n = B_n = C_n, \\ A_k = B_k = C_k = A_l = B_l = C_l, \end{aligned} \tag{6.3}$$

and

$$\begin{aligned} \sin\lambda_m = 0 = \sin\lambda_n, \\ \sin\lambda_k = 0 = \sin\lambda_l, \end{aligned} \tag{6.4}$$

so that, $\lambda_m = m\pi$ and $\lambda_n = n\pi$, where $(m, n = 1, 2, \dots)$, $\lambda_k = k\pi$, $\lambda_l = l\pi$. Following the same procedure from the generalized equation, the resulting variational equation on applying Struble's technique gives

$$\begin{aligned} &- 2\omega_{m,n}A(m,n;t)\sin[\omega_{m,n}t - \alpha_{m,n}(m,n;t)] + 2\omega_{m,n}A(m,n;t)\dot{\alpha}_{m,n} \\ &\cos[\omega_{m,n}t - \alpha_{m,n}(m,n;t)] - \lambda\left[l_x l_y \sin^2\frac{n\pi s}{l_y} + \frac{l_x l_y}{4}\right]\omega_{m,n}^2 \\ &A(m,n;t)\cos[\omega_{m,n}t - \alpha_{m,n}(m,n;t)] - \lambda\frac{v^2m^2\pi^2}{l_x^2}\left[l_x l_y \sin^2\frac{n\pi s}{l_y} + \frac{l_x l_y}{4}\right] \\ &A(m,n;t)\cos[\omega_{m,n}t - \alpha_{m,n}(m,n;t)] \end{aligned} \tag{6.5}$$

On further simplification,

$$\omega_{m,n}A(m,n;t) = 0 \tag{6.6}$$

and

$$-2\omega_{m,n}A(m,n;t)\dot{\alpha}_{m,n} - \lambda l_x l_y \omega_{m,n}^2\left[\sin^2\frac{n\pi s}{l_y} + \frac{1}{4} + \frac{v^2m^2\pi^2}{l_x^2\omega_{m,n}^2}\left(\sin^2\frac{n\pi s}{l_y} + \frac{1}{4}\right)\right] = 0. \tag{6.7}$$

Therefore the modified frequency corresponding to the frequency of system, due to the presence of moving mass, will be

$$\phi_{m,n} = \omega_{m,n}\left[1 - \frac{\lambda l_x l_y}{2}\left(\sin^2\frac{n\pi s}{l_y} + \frac{1}{4} + \frac{v^2m^2\pi^2}{l_x^2\omega_{m,n}^2}\left(\sin^2\frac{n\pi s}{l_y} + \frac{1}{4}\right)\right)\right] = 0, \tag{6.8}$$

and the reduced pair of second order coupled differential equations is

$$\begin{aligned} \phi_{m,n}(m,n;t) + \psi_{m,n}^2\phi_{m,n}(m,n;t) + \left[\frac{K_1}{\mu}\phi_{m,n}(m,n;t) - \frac{K_1}{\mu}\beta_{m,n}(m,n;t)\right] \\ = \frac{\epsilon_1 l_x l_y}{1 + \epsilon_1 R_a t}\sin\frac{m\pi vt}{l_x}\frac{n\pi s}{l_y} \end{aligned} \tag{6.9}$$

and

$$\ddot{\beta}_{m,n}(m,n;t) + \omega_{m,n}^2\phi_{m,n}(m,n;t) + \left[\frac{K_1}{\mu}\phi_{m,n}(m,n;t) - \frac{K_1}{\mu}\beta_{m,n}(m,n;t)\right] = 0. \tag{6.10}$$

Applying DTM on (6.6) and (6.9) above gives a resulting recurrence series relation as

$$\begin{aligned}\phi_{m,n}(k+2) =& \frac{1}{(k+1)(k+2)}\left[\epsilon_1 l_x l_y g \sin\frac{n\pi s}{l_y}\left(\frac{1}{k!}\left(\frac{n\pi s}{l_y}\right)\right)^k \sin\left(\frac{k\pi}{2}\right)\right.\\ &\left. - \psi_{m,n}^2\phi_{m,n}(k) - \frac{K_1}{\mu}\phi_{m,n}(k) - \frac{K_1}{\mu}\beta_{m,n}(k)\right]\end{aligned} \tag{6.11}$$

and

$$\begin{aligned}\beta_{m,n}(k+2) =& \frac{1}{(k+1)(k+2)}\\ &\left\{-\omega_{m,n}^2\beta_{m,n}(k) - \frac{K_1}{\mu}\beta_{m,n}(k) + \frac{K_1}{\mu}\phi_{m,n}(k)\right\}.\end{aligned} \tag{6.12}$$

7. Concluding remarks

The numerical results for the dynamic response of $W_1(x,y;t)$ and $W_2(x,y;t)$ were generated and presented in plotted curves with the aid of mathematical software (MAPLE V). The following values were used in the computation: $l_x = 0.457$, $l_y = 0.914$, $D = 5751$, $\mu = 0.075$, $M_l = 2$, $x = 0.2285$, $y = 0.457$, $K = 10$, $g = 10$, $s = 0.4$, $\pi = \frac{22}{7}$, $t = 0.5$, $\epsilon_1 = 0.5$ $h = 0.12$.

Figure 2 shows the deflection profile of a finite simply supported non-mindlins plate, due to moving loads, where the amplitude of the plates increases as the speed increases at different times t by varying K (layer stiffness) (i.e., $K = 40$, $K = 400$, $K = 4000$).

Figure 2.

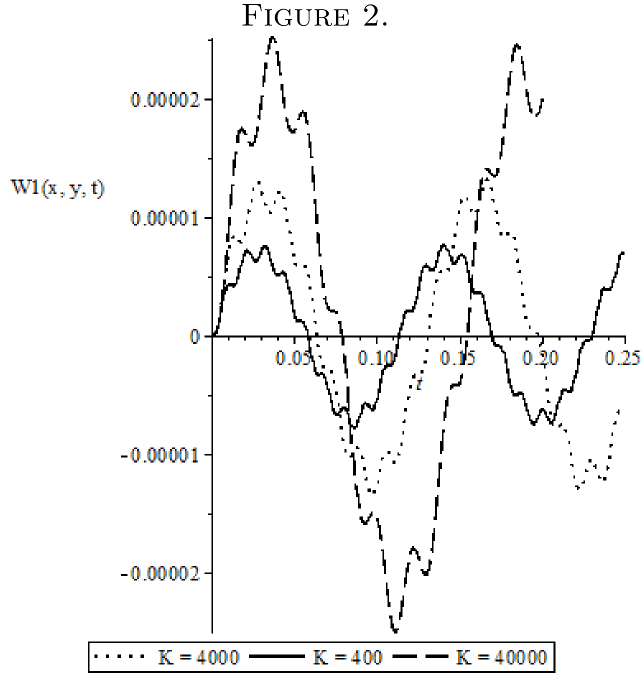


FIGURE 3. deflection profile of a finite simply supported plate. The amplitude increases with time t at $v = 5$

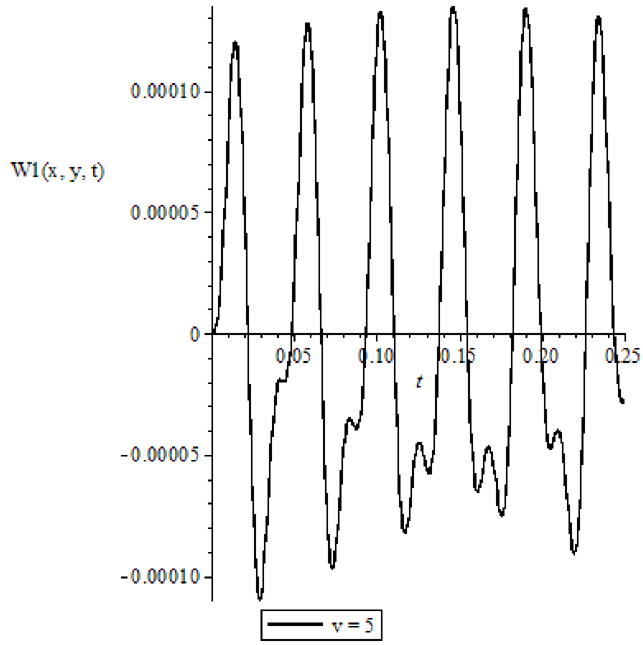


FIGURE 4. deflection profile of a finite simply supported plate. The amplitude increases with time t at $v = 7$

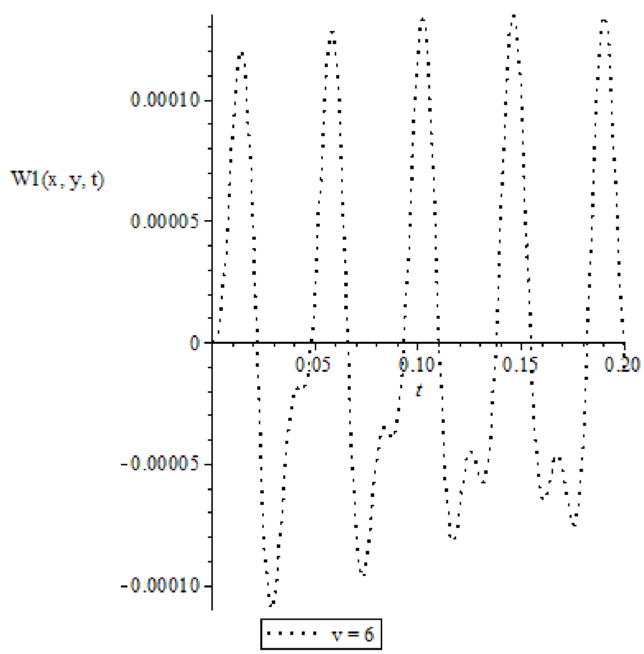
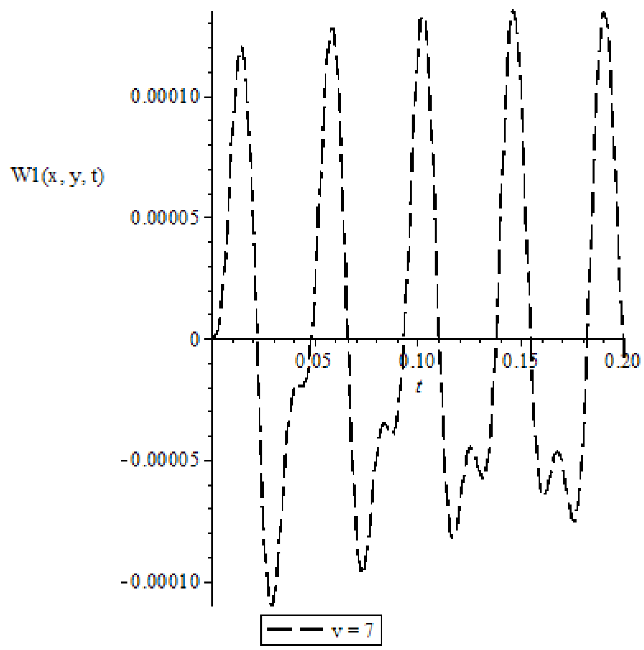


FIGURE 5. Deflection profile of a finite simply supported plate. The amplitude increases with time t at $v = 6$



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